KEYWORD: MG7 ChProj

Seeing is Disbelieving!
Many optical illusions are based on parallel and perpendicular lines. You can use these types of lines to create your own optical illusions.
**Conditional Statements**
Identify the hypothesis and conclusion of each conditional.
6. If \( E \) is on \( 
\overrightarrow{AC} \), then \( E \) lies in plane \( \mathcal{P} \).
7. If \( A \) is not in plane \( \mathcal{Q} \), then \( A \) is not on \( 
\overrightarrow{BD} \).
8. If plane \( \mathcal{P} \) and plane \( \mathcal{Q} \) intersect, then they intersect in a line.

**Name and Classify Angles**
Name and classify each angle.
9. \( \angle GJK \)
10. \( \angle MKL \)
11. \( \angle PQR \)
12. \( \angle RST \)

**Angle Relationships**
Give an example of each angle pair.
13. vertical angles
14. adjacent angles
15. complementary angles
16. supplementary angles

**Evaluate Expressions**
Evaluate each expression for the given value of the variable.
17. \( 4x + 9 \) for \( x = 31 \)
18. \( 6x - 16 \) for \( x = 43 \)
19. \( 97 - 3x \) for \( x = 20 \)
20. \( 5x + 3x + 12 \) for \( x = 17 \)

**Solve Multi-Step Equations**
Solve each equation for \( x \).
21. \( 4x + 8 = 24 \)
22. \( 2 = 2x - 8 \)
23. \( 4x + 3x + 6 = 90 \)
24. \( 21x + 13 + 14x - 8 = 180 \)
Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>alternate exterior angles</td>
<td>ángulos alternos externos</td>
</tr>
<tr>
<td>alternate interior angles</td>
<td>ángulos alternos internos</td>
</tr>
<tr>
<td>corresponding angles</td>
<td>ángulos correspondientes</td>
</tr>
<tr>
<td>parallel lines</td>
<td>líneas paralelas</td>
</tr>
<tr>
<td>perpendicular bisector</td>
<td>mediatriz</td>
</tr>
<tr>
<td>perpendicular lines</td>
<td>líneas perpendiculares</td>
</tr>
<tr>
<td>same-side interior angles</td>
<td>ángulos internos del mismo lado</td>
</tr>
<tr>
<td>slope</td>
<td>pendiente</td>
</tr>
<tr>
<td>transversal</td>
<td>transversal</td>
</tr>
</tbody>
</table>

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, answer the following questions. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The root *trans-* means “across.” What do you think a *transversal* of two lines does?

2. The *slope* of a mountain trail describes the steepness of the climb. What might the *slope* of a line describe?

3. What does the word *corresponding* mean? What do you think the term *corresponding angles* means?

4. What does the word *interior* mean? What might the phrase “interior of a pair of lines” describe? The word *alternate* means “to change from one to another.” If two lines are crossed by a third line, where do you think a pair of *alternate interior angles* might be?
Study Strategy: Take Effective Notes
Taking effective notes is an important study strategy. The Cornell system of note taking is a good way to organize and review main ideas. In the Cornell system, the paper is divided into three main sections. The note-taking column is where you take notes during lecture. The cue column is where you write questions and key phrases as you review your notes. The summary area is where you write a brief summary of the lecture.

Try This
1. Research and write a paragraph describing the Cornell system of note taking. Describe how you can benefit from using this type of system.
2. In your next class, use the Cornell system of note taking. Compare these notes to your notes from a previous lecture.
**Objectives**
Identify parallel, perpendicular, and skew lines.
Identify the angles formed by two lines and a transversal.

**Vocabulary**
- parallel lines
- perpendicular lines
- skew lines
- parallel planes
- transversal
- corresponding angles
- alternate interior angles
- alternate exterior angles
- same-side interior angles

**Who uses this?**
Card architects use playing cards to build structures that contain parallel and perpendicular planes.

Bryan Berg uses cards to build structures like the one at right. In 1992, he broke the Guinness World Record for card structures by building a tower 14 feet 6 inches tall. Since then, he has built structures more than 25 feet tall.

**Parallel, Perpendicular, and Skew Lines**

**Parallel lines** (ǁ) are coplanar and do not intersect. In the figure, \( \overline{AB} \parallel \overline{EF} \), and \( \overline{EG} \parallel \overline{FH} \).

**Perpendicular lines** (\( \perp \)) intersect at 90° angles. In the figure, \( \overline{AB} \perp \overline{AE} \), and \( \overline{EG} \perp \overline{GH} \).

**Skew lines** are not coplanar. Skew lines are not parallel and do not intersect. In the figure, \( \overline{AB} \) and \( \overline{EG} \) are skew.

**Parallel planes** are planes that do not intersect. In the figure, plane \( \overline{ABE} \parallel \) plane \( \overline{CDG} \).

**EXAMPLE 1**
Identify each of the following.

**A** a pair of parallel segments
\( \overline{KN} \parallel \overline{PS} \)

**B** a pair of skew segments
\( \overline{LM} \) and \( \overline{RS} \) are skew.

**C** a pair of perpendicular segments
\( \overline{MR} \perp \overline{RS} \)

**D** a pair of parallel planes
plane \( \overline{KPS} \parallel \) plane \( \overline{LQR} \)

**CHECK IT OUT!**
Identify each of the following.

1a. a pair of parallel segments
1b. a pair of skew segments
1c. a pair of perpendicular segments
1d. a pair of parallel planes
**Term**

**Example**

A **transversal** is a line that intersects two coplanar lines at two different points. The transversal \( t \) and the other two lines \( r \) and \( s \) form eight angles.

**Corresponding angles** lie on the same side of the transversal \( t \), on the same sides of lines \( r \) and \( s \).

\[ \angle 1 \text{ and } \angle 5 \]

**Alternate interior angles** are nonadjacent angles that lie on opposite sides of the transversal \( t \), between lines \( r \) and \( s \).

\[ \angle 3 \text{ and } \angle 6 \]

**Alternate exterior angles** lie on opposite sides of the transversal \( t \), outside lines \( r \) and \( s \).

\[ \angle 1 \text{ and } \angle 8 \]

**Same-side interior angles or consecutive interior angles** lie on the same side of the transversal \( t \), between lines \( r \) and \( s \).

\[ \angle 3 \text{ and } \angle 5 \]

---

**Example 2**

Classifying Pairs of Angles

Give an example of each angle pair.

A **corresponding angles**

\[ \angle 4 \text{ and } \angle 8 \]

B **alternate interior angles**

\[ \angle 4 \text{ and } \angle 6 \]

C **alternate exterior angles**

\[ \angle 2 \text{ and } \angle 8 \]

D **same-side interior angles**

\[ \angle 4 \text{ and } \angle 5 \]

---

**Example 3**

Identifying Angle Pairs and Transversals

Identify the transversal and classify each angle pair.

A \[ \angle 1 \text{ and } \angle 5 \]

transversal: \( n \); alternate interior angles

B \[ \angle 3 \text{ and } \angle 6 \]

transversal: \( m \); corresponding angles

C \[ \angle 1 \text{ and } \angle 4 \]

transversal: \( \ell \); alternate exterior angles

3. Identify the transversal and classify the angle pair \( \angle 2 \) and \( \angle 5 \) in the diagram above.
**THINK AND DISCUSS**

1. Compare perpendicular and intersecting lines.
2. Describe the positions of two alternate exterior angles formed by lines \( m \) and \( n \) with transversal \( p \).
3. **GET ORGANIZED** Copy the diagram and graphic organizer. In each box, list all the angle pairs of each type in the diagram.

![Diagram of Pairs of Angles](image)

**GUIDED PRACTICE**

1. **Vocabulary** blanks are located on opposite sides of a transversal, between the two lines that intersect the transversal. (corresponding angles, alternate interior angles, alternate exterior angles, or same-side interior angles)

**SEE EXAMPLE 1**

p. 146

Identify each of the following.

2. one pair of perpendicular segments
3. one pair of skew segments
4. one pair of parallel segments
5. one pair of parallel planes

**SEE EXAMPLE 2**

p. 147

Give an example of each angle pair.

6. alternate interior angles
7. alternate exterior angles
8. corresponding angles
9. same-side interior angles

**SEE EXAMPLE 3**

p. 147

Identify the transversal and classify each angle pair.

10. \( \angle 1 \) and \( \angle 2 \)
11. \( \angle 2 \) and \( \angle 3 \)
12. \( \angle 2 \) and \( \angle 4 \)
13. \( \angle 4 \) and \( \angle 5 \)
Identify each of the following.
14. one pair of parallel segments
15. one pair of skew segments
16. one pair of perpendicular segments
17. one pair of parallel planes

Give an example of each angle pair.
18. same-side interior angles
19. alternate exterior angles
20. corresponding angles
21. alternate interior angles

Identify the transversal and classify each angle pair.
22. \( \angle 2 \) and \( \angle 3 \)
23. \( \angle 4 \) and \( \angle 5 \)
24. \( \angle 2 \) and \( \angle 4 \)
25. \( \angle 1 \) and \( \angle 2 \)

26. **Sports** A football player runs across the 30-yard line at an angle. He continues in a straight line and crosses the goal line at the same angle. Describe two parallel lines and a transversal in the diagram.

Name the type of angle pair shown in each letter.
27. F
28. Z
29. C

**Entertainment** Use the following information for Exercises 30–32.
In an Ames room, the floor is tilted and the back wall is closer to the front wall on one side.
30. Name a pair of parallel segments in the diagram.
31. Name a pair of skew segments in the diagram.
32. Name a pair of perpendicular segments in the diagram.
33. This problem will prepare you for the Multi-Step Test Prep on p 180. Buildings that are tilted like the one shown are sometimes called mystery spots.
   a. Name a plane parallel to plane KLP, a plane parallel to plane KNP, and a plane parallel to KLM.
   b. In the diagram, $QR$ is a transversal to $PQ$ and $RS$. What type of angle pair is $\angle PQR$ and $\angle QRS$?

34. **Critical Thinking** Line $\ell$ is contained in plane $P$ and line $m$ is contained in plane $Q$. If $P$ and $Q$ are parallel, what are the possible classifications of $\ell$ and $m$? Include diagrams to support your answer.

Use the diagram for Exercises 35–40.

35. Name a pair of alternate interior angles with transversal $n$.
36. Name a pair of same-side interior angles with transversal $\ell$.
37. Name a pair of corresponding angles with transversal $m$.
38. Identify the transversal and classify the angle pair for $\angle 3$ and $\angle 7$.
39. Identify the transversal and classify the angle pair for $\angle 5$ and $\angle 8$.
40. Identify the transversal and classify the angle pair for $\angle 1$ and $\angle 6$.

41. **Aviation** Describe the type of lines formed by two planes when flight 1449 is flying from San Francisco to Atlanta at 32,000 feet and flight 2390 is flying from Dallas to Chicago at 28,000 feet.

42. **Multi-Step** Draw line $p$, then draw two lines $m$ and $n$ that are both perpendicular to $p$. Make a conjecture about the relationship between lines $m$ and $n$.

43. **Write About It** Discuss a real-world example of skew lines. Include a sketch.

44. Which pair of angles in the diagram are alternate interior angles?
   - A $\angle 1$ and $\angle 5$
   - B $\angle 2$ and $\angle 6$
   - C $\angle 7$ and $\angle 5$
   - D $\angle 2$ and $\angle 3$

45. How many pairs of corresponding angles are in the diagram?
   - E 2
   - G 4
   - F 8
   - H 16
46. Which type of lines are NOT represented in the diagram?
   - Parallel lines
   - Skew lines
   - Intersecting lines
   - Perpendicular lines

47. For two lines and a transversal, \( \angle 1 \) and \( \angle 8 \) are alternate exterior angles, and \( \angle 1 \) and \( \angle 5 \) are corresponding angles. Classify the angle pair \( \angle 5 \) and \( \angle 8 \).
   - Vertical angles
   - Alternate interior angles
   - Adjacent angles
   - Same-side interior angles

48. Which angles in the diagram are NOT corresponding angles?
   - \( \angle 1 \) and \( \angle 5 \)
   - \( \angle 4 \) and \( \angle 8 \)
   - \( \angle 2 \) and \( \angle 6 \)
   - \( \angle 2 \) and \( \angle 7 \)

**CHALLENGE AND EXTEND**

Name all the angle pairs of each type in the diagram. Identify the transversal for each pair.

49. corresponding
50. alternate interior
51. alternate exterior
52. same-side interior

53. Multi-Step Draw two lines and a transversal such that \( \angle 1 \) and \( \angle 3 \) are corresponding angles, \( \angle 1 \) and \( \angle 2 \) are alternate interior angles, and \( \angle 3 \) and \( \angle 4 \) are alternate exterior angles. What type of angle pair is \( \angle 2 \) and \( \angle 4 \)?

54. If the figure shown is folded to form a cube, which faces of the cube will be parallel?

**SPIRAL REVIEW**

Evaluate each function for \( x = -1, 0, 1, 2, \) and \( 3 \). (Previous course)

55. \( y = 4x^2 - 7 \)
56. \( y = -2x^2 + 5 \)
57. \( y = (x + 3)(x - 3) \)

Find the circumference and area of each circle. Use the \( \pi \) key on your calculator and round to the nearest tenth. (Lesson 1-5)

58.

59.

Write a justification for each statement, given that \( \angle 1 \) and \( \angle 3 \) are right angles. (Lesson 2-6)

60. \( \angle 1 \cong \angle 3 \)
61. \( m\angle 1 + m\angle 2 = 180^\circ \)
62. \( \angle 2 \cong \angle 4 \)
Example 1

Solve for \( x \) and \( y \).

Since the lines are perpendicular, all of the angles are right angles. To write two equations, you can set each expression equal to \( 90^\circ \).

\[
(3x + 2y)^\circ = 90^\circ, \quad (6x - 2y)^\circ = 90^\circ
\]

**Step 1**

\[
3x + 2y = 90 \\
6x - 2y = 90
\]

*Write the system so that like terms are under one another.*

**Step 2**

\[
x = 20
\]

*Add like terms on each side of the equations. The \( y \)-term has been eliminated.*

*Divide both sides by 9 to solve for \( x \).*

**Step 3**

\[
3(20) + 2y = 90 \\
60 + 2y = 90
\]

*Write one of the original equations. Substitute 20 for \( x \).*

*Simplify.*

\[
2y = 30
\]

*Subtract 60 from both sides.*

\[
y = 15
\]

*Divide by 2 on both sides.*

**Step 4**

\[
(20, 15)
\]

*Write the solution as an ordered pair.*

**Step 5**

Check the solution by substituting 20 for \( x \) and 15 for \( y \) in the original equations.

\[
\begin{array}{ccc}
3x + 2y &=& 90 \\
6x - 2y &=& 90 \\
3(20) + 2(15) &=& 90 \\
60 + 2(15) &=& 90 \\
60 + 30 &=& 90 \\
120 - 30 &=& 90
\end{array}
\]

\[
90 \quad 90 \quad \checkmark 
\]

In some cases, before you can do Step 1 you will need to multiply one or both of the equations by a number so that you can eliminate a variable.
Example 2

Solve for \(x\) and \(y\).

\[
\begin{align*}
(2x + 4y)^\circ &= 72^\circ & \text{Vertical Angles Theorem} \\
(5x + 2y)^\circ &= 108^\circ & \text{Linear Pair Theorem}
\end{align*}
\]

The equations cannot be added or subtracted to eliminate a variable. Multiply the second equation by \(-2\) to get opposite \(y\)-coefficients.

\[
5x + 2y = 108 \rightarrow -2(5x + 2y) = -2(108) \rightarrow -10x - 4y = -216
\]

Step 1

\[
\begin{align*}
2x + 4y &= 72 \\
-10x - 4y &= -216
\end{align*}
\]

Write the system so that like terms are under one another.

Step 2

\[-8x = -144\]

Add like terms on both sides of the equations. The \(y\)-term has been eliminated.

\[x = 18\]

Divide both sides by \(-8\) to solve for \(x\).

Step 3

\[
2x + 4y = 72
\]

Write one of the original equations.

\[2(18) + 4y = 72\]

Substitute 18 for \(x\).

\[36 + 4y = 72\]

Simplify.

\[4y = 36\]

Subtract 36 from both sides.

\[y = 9\]

Divide by 4 on both sides.

Step 4

\[(18, 9)\]

Write the solution as an ordered pair.

Step 5

Check the solution by substituting 18 for \(x\) and 9 for \(y\) in the original equations.

\[
\begin{array}{c|c|c}
2x + 4y &= 72 & 5x + 2y = 108 \\
3(18) + 4(9) &= 72 & 5(18) + 2(9) &= 108 \\
36 + 36 &= 72 & 90 + 18 &= 108 \\
72 &= 72 & 108 &= 108
\end{array}
\]

Try This

Solve for \(x\) and \(y\).

1.

\[
\begin{align*}
(10x + 4y)^\circ &= 72^\circ \\
(26x - 4y)^\circ &= 45^\circ
\end{align*}
\]

2.

\[
\begin{align*}
(3x + 3y)^\circ &= 45^\circ \\
(-3x + 17y)^\circ &= 90^\circ
\end{align*}
\]

3.

\[
\begin{align*}
(18x + 6y)^\circ &= 72^\circ \\
(6x + 10y)^\circ &= 36^\circ
\end{align*}
\]

4.

\[
\begin{align*}
(32x + 2y)^\circ &= 72^\circ \\
(19x + 4y)^\circ &= 108^\circ
\end{align*}
\]
Explore Parallel Lines and Transversals

Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

Use with Lesson 3-2

**Activity**

1. Construct a line and label two points on the line \( A \) and \( B \).

2. Create point \( C \) not on \( AB \). Construct a line parallel to \( AB \) through point \( C \). Create another point on this line and label it \( D \).

3. Create two points outside the two parallel lines and label them \( E \) and \( F \). Construct transversal \( EF \). Label the points of intersection \( G \) and \( H \).

4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point \( E \) or \( F \) and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

<table>
<thead>
<tr>
<th>Angle</th>
<th>( \angle GAE )</th>
<th>( \angle BGE )</th>
<th>( \angle AGH )</th>
<th>( \angle BGH )</th>
<th>( \angle CHG )</th>
<th>( \angle DHG )</th>
<th>( \angle CHF )</th>
<th>( \angle DHF )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Try This**

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.

2. Repeat steps in the previous problem for alternate interior angles, alternate exterior angles, and same-side interior angles.

3. Try dragging point \( C \) to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?
### Who uses this?

Piano makers use parallel strings for the higher notes. The longer strings used to produce the lower notes can be viewed as transversals. (See Example 3.)

When parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary.

### Postulate 3-2-1  
**Corresponding Angles Postulate**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
</table>
| If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent. | ![Diagram of angles](image) | \( \angle 1 \cong \angle 3 \)  
\( \angle 2 \cong \angle 4 \)  
\( \angle 5 \cong \angle 7 \)  
\( \angle 6 \cong \angle 8 \) |

### Example 1  
**Using the Corresponding Angles Postulate**

Find each angle measure.

**A**  
\[ m\angle ABC = 80^\circ \]  
\[ m\angle ABC = 80^\circ \]  
**B**  
\[ m\angle DEF = (2x - 45)^\circ = (x + 30)^\circ \]  
\[ x = 75 \]  
\[ m\angle DEF = x + 30 \]  
\[ m\angle DEF = 105^\circ \]

### Check it out!

1. Find \( m\angle QRS \).

Remember that postulates are statements that are accepted without proof. Since the Corresponding Angles Postulate is given as a postulate, it can be used to prove the next three theorems.
THEOREM

3-2-2 Alternate Interior Angles Theorem
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

\[ \angle 1 \cong \angle 3 \]
\[ \angle 2 \cong \angle 4 \]

3-2-3 Alternate Exterior Angles Theorem
If two parallel lines are cut by a transversal, then the two pairs of alternate exterior angles are congruent.

\[ \angle 5 \cong \angle 7 \]
\[ \angle 6 \cong \angle 8 \]

3-2-4 Same-Side Interior Angles Theorem
If two parallel lines are cut by a transversal, then the two pairs of same-side interior angles are supplementary.

\[ m\angle 1 + m\angle 4 = 180^\circ \]
\[ m\angle 2 + m\angle 3 = 180^\circ \]

You will prove Theorems 3-2-3 and 3-2-4 in Exercises 25 and 26.

PROOF

Alternate Interior Angles Theorem

Given: \( \ell \parallel m \)
Prove: \( \angle 2 \cong \angle 3 \)
Proof:

\[ \angle 1 \cong \angle 3 \]
Corr. \( \triangle \) Post.

\[ \angle 2 \cong \angle 1 \]
Vert. \( \triangle \) Thm.

Finding Angle Measures

Find each angle measure.

**A** \( m\angle EDF \)

\[ x = 125 \]
\[ m\angle EDF = 125^\circ \]

**B** \( m\angle TUS \)

\[ 13x^\circ + 23x^\circ = 180^\circ \]
\[ 36x = 180 \]
\[ x = 5 \]
\[ m\angle TUS = 23(5) = 115^\circ \]

**CHECK IT OUT!**

2. Find \( m\angle ABD \).
### Music Application

The treble strings of a grand piano are parallel. Viewed from above, the bass strings form transversals to the treble strings. Find \(x\) and \(y\) in the diagram.

By the Alternate Exterior Angles Theorem, \((25x + 5y)° = 125°\).

By the Corresponding Angles Postulate, \((25x + 4y)° = 120°\).

\[
25x + 5y = 125
\]

\[
(25x + 4y + 120)
\]

\[
y = 5
\]

\[
25x + 5(5) = 125
\]

\[
x = 4, \quad y = 5
\]

3. Find the measures of the acute angles in the diagram.

### THINK AND DISCUSS

1. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

2. **GET ORGANIZED** Copy the diagram and graphic organizer. Complete the graphic organizer by explaining why each of the three theorems is true.
GUIDED PRACTICE

See example 1  p. 155

Find each angle measure.
1. m∠JKL
   \[ \angle LJK = 127° \]

3. m∠1
   \[ \angle 1 \]

5. Safety The railing of a wheelchair ramp is parallel to the ramp. Find x and y in the diagram.

See example 2  p. 156

4. m∠CBY

See example 3  p. 157

2. m∠BEF
   \[ \angle BEF = (7x - 14)° \]

6. m∠KLM
   \[ \angle KLM = 115° \]

8. m∠ABC
   \[ \angle ABC = x° \]

10. m∠PQR
    \[ \angle PQR = (3n - 45)° \]

PRACTICE AND PROBLEM SOLVING

Find each angle measure.
6. m∠KLM
    \[ \angle KLM = 115° \]

8. m∠ABC
    \[ \angle ABC = x° \]

10. m∠PQR
    \[ \angle PQR = (3n - 45)° \]

7. m∠VYX
    \[ \angle VYX = (2a + 50)° \]

9. m∠EFG
    \[ \angle EFG = 13x° \]

11. m∠STU
    \[ \angle STU = (4x - 14)° \]
12. **Parking** In the parking lot shown, the lines that mark the width of each space are parallel.
   \[ \angle 1 = (2x - 3y)^\circ \]
   \[ \angle 2 = (x + 3y)^\circ \]
   Find \( x \) and \( y \).

Find each angle measure. Justify each answer with a postulate or theorem.

13. \( \angle 1 \)
14. \( \angle 2 \)
15. \( \angle 3 \)
16. \( \angle 4 \)
17. \( \angle 5 \)
18. \( \angle 6 \)
19. \( \angle 7 \)

**Algebra** State the theorem or postulate that is related to the measures of the angles in each pair. Then find the angle measures.

20. \( \angle 1 = (7x + 15)^\circ \), \( \angle 2 = (10x - 9)^\circ \)
21. \( \angle 3 = (23x + 11)^\circ \), \( \angle 4 = (14x + 21)^\circ \)
22. \( \angle 4 = (37x - 15)^\circ \), \( \angle 5 = (44x - 29)^\circ \)
23. \( \angle 1 = (6x + 24)^\circ \), \( \angle 4 = (17x - 9)^\circ \)

24. **Architecture** The Luxor Hotel in Las Vegas, Nevada, is a 30-story pyramid. The hotel uses an elevator called an inclinator to take people up the side of the pyramid. The inclinator travels at a 39° angle. Which theorem or postulate best illustrates the angles formed by the path of the inclinator and each parallel floor? (Hint: Draw a picture.)

25. **Complete the two-column proof of the Alternate Exterior Angles Theorem.**

   **Given:** \( l \parallel m \)
   **Prove:** \( \angle 1 \cong \angle 2 \)
   **Proof:**

   \[
   \begin{array}{|c|c|}
   \hline
   \text{Statements} & \text{Reasons} \\
   \hline
   1. \ l \parallel m & 1. \ \text{Given} \\
   2. \text{a. } ? & 2. \ \text{Vert. } \triangle \text{ Thm.} \\
   3. \ \angle 3 \cong \angle 2 & 3. \text{b. } ? \\
   4. \ \text{c. } ? & 4. \text{d. } ? \\
   \hline
   \end{array}
   \]

26. **Write a paragraph proof of the Same-Side Interior Angles Theorem.**

   **Given:** \( r \parallel s \)
   **Prove:** \( \angle 1 + \angle 2 = 180^\circ \)

   Draw the given situation or tell why it is impossible.

27. Two parallel lines are intersected by a transversal so that the corresponding angles are supplementary.

28. Two parallel lines are intersected by a transversal so that the same-side interior angles are complementary.
29. This problem will prepare you for the Multi-Step Test Prep on page 180.

In the diagram, which represents the side view of a mystery spot, \( \angle SRT = 25^\circ \). \( \overrightarrow{RT} \) is a transversal to \( \overrightarrow{PS} \) and \( \overrightarrow{QR} \).

a. What type of angle pair is \( \angle QRT \) and \( \angle STR \)?

b. Find \( \angle STR \). Use a theorem or postulate to justify your answer.

30. **Land Development** A piece of property lies between two parallel streets as shown. \( \angle 1 = (2x + 6)^\circ \), and \( \angle 2 = (3x + 9)^\circ \).

What is the relationship between the angles? What are their measures?

31. **ERROR ANALYSIS** In the figure, \( \angle ABC = (15x + 5)^\circ \), and \( \angle BCD = (10x + 25)^\circ \).

Which value of \( \angle BCD \) is incorrect? Explain.

32. **Critical Thinking** In the diagram, \( \ell \parallel m \).

Explain why \( \frac{x}{y} = 1 \).

33. **Write About It** Suppose that lines \( \ell \) and \( m \) are intersected by transversal \( p \). One of the angles formed by \( \ell \) and \( p \) is congruent to every angle formed by \( m \) and \( p \). Draw a diagram showing lines \( \ell \), \( m \), and \( p \), mark any congruent angles that are formed, and explain what you know is true.

34. \( \angle RST = (x + 50)^\circ \), and \( \angle STU = (3x + 20)^\circ \).

Find \( m \angle RV T \).

- \( \text{A} \) 15°
- \( \text{B} \) 27.5°
- \( \text{C} \) 65°
- \( \text{D} \) 77.5°
35. For two parallel lines and a transversal, \( m \angle 1 = 83\degree \). For which pair of angle measures is the sum the least?
   - \( \text{C} \) \( \angle 1 \) and a corresponding angle
   - \( \text{G} \) \( \angle 1 \) and a same-side interior angle
   - \( \text{H} \) \( \angle 1 \) and its supplement
   - \( \text{J} \) \( \angle 1 \) and its complement

36. **Short Response** Given \( a \parallel b \) with transversal \( t \), explain why \( \angle 1 \) and \( \angle 3 \) are supplementary.

\[
\begin{align*}
\angle 1 & \parallel \angle 3 \\
\angle 1 & \parallel \angle 3
\end{align*}
\]

**CHALLENGE AND EXTEND**

**Multi-Step** Find \( m \angle 1 \) in each diagram. (Hint: Draw a line parallel to the given parallel lines.)

37.

\[
\begin{align*}
\angle 1 & = 145\degree \\
\angle 2 & = 40\degree
\end{align*}
\]

38.

\[
\begin{align*}
\angle 1 & = 105\degree \\
\angle 2 & = 80\degree
\end{align*}
\]

39. Find \( x \) and \( y \) in the diagram. Justify your answer.

40. Two lines are parallel. The measures of two corresponding angles are \( a\degree \) and \( 2b\degree \), and the measures of two same-side interior angles are \( a\degree \) and \( b\degree \). Find the value of \( a \).

**SPIRAL REVIEW**

If the first quantity increases, tell whether the second quantity is likely to increase, decrease, or stay the same. (Previous course)

41. time in years and average cost of a new car

42. age of a student and length of time needed to read 500 words

Use the Law of Syllogism to draw a conclusion from the given information. (Lesson 2-3)

43. If two angles form a linear pair, then they are supplementary. If two angles are supplementary, then their measures add to 180\degree. \( \angle 1 \) and \( \angle 2 \) form a linear pair.

44. If a figure is a square, then it is a rectangle. If a figure is a rectangle, then its sides are perpendicular. Figure \( ABCD \) is a square.

Give an example of each angle pair. (Lesson 3-1)

45. alternate interior angles

46. alternate exterior angles

47. same-side interior angles
Recall that the converse of a theorem is found by exchanging the hypothesis and conclusion. The converse of a theorem is not automatically true. If it is true, it must be stated as a postulate or proved as a separate theorem.

**THEOREM**

If two coplanar lines are cut by a transversal so that a pair of corresponding angles are congruent, then the two lines are parallel.

\[ \angle 1 \cong \angle 2 \]

\[ m \parallel n \]

**Example 1**

Using the Converse of the Corresponding Angles Postulate

Use the Converse of the Corresponding Angles Postulate and the given information to show that \( \ell \parallel m \).

**A**

\[ \angle 1 \cong \angle 5 \]
\[ \angle 1 \cong \angle 5 \]
\[ \angle 1 \text{ and } \angle 5 \text{ are corresponding angles.} \]
\[ \ell \parallel m \text{ Conv. of Corr. } \angle s \text{ Post.} \]

**B**

\[ m\angle 4 = (2x + 10)^\circ, m\angle 8 = (3x - 55)^\circ, x = 65 \]
\[ m\angle 4 = 2(65) + 10 = 140 \text{ Substitute 65 for } x. \]
\[ m\angle 8 = 3(65) - 55 = 140 \text{ Substitute 65 for } x. \]
\[ m\angle 4 = m\angle 8 \text{ Trans. Prop. of Equality} \]
\[ \angle 4 \cong \angle 8 \text{ Def. of } \cong \angle \]
\[ \ell \parallel m \text{ Conv. of Corr. } \angle \text{ Post.} \]

**Check It Out!**

Use the Converse of the Corresponding Angles Postulate and the given information to show that \( \ell \parallel m \).

1a. \( m\angle 1 = m\angle 3 \)
1b. \( m\angle 7 = (4x + 25)^\circ, m\angle 5 = (5x + 12)^\circ, x = 13 \)
The Converse of the Corresponding Angles Postulate is used to construct parallel lines. The Parallel Postulate guarantees that for any line \( \ell \), you can always construct a parallel line through a point that is not on \( \ell \).

**Construction Parallel Lines**

1. Draw a line \( \ell \) and a point \( P \) that is not on \( \ell \).
2. Draw a line \( m \) through \( P \) that intersects \( \ell \). Label the angle 1.
3. Construct an angle congruent to \( \angle 1 \) at \( P \). By the converse of the Corresponding Angles Postulate, \( \ell \parallel n \).

**Theorems Proving Lines Parallel**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-3-3 Converse of the Alternate Interior Angles Theorem</td>
<td>( \angle 1 \cong \angle 2 )</td>
<td>( m \parallel n )</td>
</tr>
<tr>
<td>3-3-4 Converse of the Alternate Exterior Angles Theorem</td>
<td>( \angle 3 \cong \angle 4 )</td>
<td>( m \parallel n )</td>
</tr>
<tr>
<td>3-3-5 Converse of the Same-Side Interior Angles Theorem</td>
<td>( m\angle 5 + m\angle 6 = 180^\circ )</td>
<td>( m \parallel n )</td>
</tr>
</tbody>
</table>

You will prove Theorems 3-3-3 and 3-3-5 in Exercises 38–39.
Converse of the Alternate Exterior Angles Theorem

Given: ∠1 ≅ ∠2
Prove: ℓ || m

Proof: It is given that ∠1 ≅ ∠2. Vertical angles are congruent, so ∠1 ≅ ∠3. By the Transitive Property of Congruence, ∠2 ≅ ∠3. So ℓ || m by the Converse of the Corresponding Angles Postulate.

Example 2

Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that r || s.

A  \( \angle 2 \equiv \angle 6 \)
\( \angle 2 \equiv \angle 6 \) \( \angle 2 \) and \( \angle 6 \) are alternate interior angles.
\( r \parallel s \) Conv. of Alt. Int. \( \triangle \) Thm.

B  \( m \angle 6 = (6x + 18)^\circ \), \( m \angle 7 = (9x + 12)^\circ \), \( x = 10 \)
\( m \angle 6 = 6x + 18 \)
\( = 6(10) + 18 = 78^\circ \) Substitute 10 for x.
\( m \angle 7 = 9x + 12 \)
\( = 9(10) + 12 = 102^\circ \) Substitute 10 for x.
\( m \angle 6 + m \angle 7 = 78^\circ + 102^\circ \)
\( = 180^\circ \) \( \angle 6 \) and \( \angle 7 \) are same-side interior angles.
\( r \parallel s \) Conv. of Same-Side Int. \( \triangle \) Thm.

Refer to the diagram above. Use the given information and the theorems you have learned to show that \( r \parallel s \).

2a. \( m \angle 4 = m \angle 8 \) 2b. \( m \angle 3 = 2x^\circ \), \( m \angle 7 = (x + 50)^\circ \), \( x = 50 \)

Example 3

Proving Lines Parallel

Given: \( \ell \parallel m \), \( \angle 1 \equiv \angle 3 \)
Prove: \( r \parallel p \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \ell \parallel m )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \equiv \angle 2 )</td>
<td>2. Corr. ( \triangle ) Post.</td>
</tr>
<tr>
<td>3. ( \angle 1 \equiv \angle 3 )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \angle 2 \equiv \angle 3 )</td>
<td>4. Trans. Prop. of ( \equiv )</td>
</tr>
<tr>
<td>5. ( r \parallel p )</td>
<td>5. Conv. of Alt. Ext. ( \triangle ) Thm.</td>
</tr>
</tbody>
</table>

3. Given: \( \angle 1 \equiv \angle 4 \), \( \angle 3 \) and \( \angle 4 \) are supplementary.

Prove: \( \ell \parallel m \)
**Sports Application**

During a race, all members of a rowing team should keep the oars parallel on each side. If \( m \angle 1 = (3x + 13) ^\circ \), \( m \angle 2 = (5x - 5) ^\circ \), and \( x = 9 \), show that the oars are parallel.

A line through the center of the boat forms a transversal to the two oars on each side of the boat.

\[ \angle 1 \text{ and } \angle 2 \text{ are corresponding angles.} \]

If \( \angle 1 \cong \angle 2 \), then the oars are parallel.

Substitute 9 for \( x \) in each expression:

\[ m \angle 1 = 3x + 13 \]
\[ = 3(9) + 13 = 40 ^\circ \]

Substitute 9 for \( x \) in each expression.

\[ m \angle 2 = 5x - 5 \]
\[ = 5(9) - 5 = 40 ^\circ \]

\[ m \angle 1 = m \angle 2, \text{ so } \angle 1 \cong \angle 2. \]

The corresponding angles are congruent, so the oars are parallel by the Converse of the Corresponding Angles Postulate.

**4. What if...?** Suppose the corresponding angles on the opposite side of the boat measure \((4y - 2) ^\circ\) and \((3y + 6) ^\circ\), where \( y = 8 \). Show that the oars are parallel.

**THINK AND DISCUSS**

1. Explain three ways of proving that two lines are parallel.

2. If you know \( m \angle 1 \), how could you use the measures of \( \angle 5, \angle 6, \angle 7, \) or \( \angle 8 \) to prove \( m \parallel n \)?

3. **GET ORGANIZED** Copy and complete the graphic organizer. Use it to compare the Corresponding Angles Postulate with the Converse of the Corresponding Angles Postulate.
GUARDPRACTICE

Use the Converse of the Corresponding Angles Postulate and the given information to show that $p \parallel q$.

1. $\angle 4 \cong \angle 5$
2. $m\angle 1 = (4x + 16)^\circ, m\angle 8 = (5x - 12)^\circ, x = 28$
3. $m\angle 4 = (6x - 19)^\circ, m\angle 5 = (3x + 14)^\circ, x = 11$

Use the theorems and given information to show that $r \parallel s$.

4. $\angle 1 \cong \angle 5$
5. $m\angle 3 + m\angle 4 = 180^\circ$
6. $\angle 3 \cong \angle 7$
7. $m\angle 4 = (13x - 4)^\circ, m\angle 8 = (9x + 16)^\circ, x = 5$
8. $m\angle 8 = (17x + 37)^\circ, m\angle 7 = (9x - 13)^\circ, x = 6$
9. $m\angle 2 = (25x + 7)^\circ, m\angle 6 = (24x + 12)^\circ, x = 5$

10. Complete the following two-column proof.

   **Given:** $\angle 1 \cong \angle 2, \angle 3 \cong \angle 1$
   **Prove:** $XY \parallel WV$
   **Proof:**
   
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2, \angle 3 \cong \angle 1$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 2 \cong \angle 3$</td>
<td>2. a. <em><strong>?</strong></em></td>
</tr>
<tr>
<td>3. b. <em><strong>?</strong></em></td>
<td>3. c. <em><strong>?</strong></em></td>
</tr>
</tbody>
</table>

11. **Architecture** In the fire escape, $m\angle 1 = (17x + 9)^\circ, m\angle 2 = (14x + 18)^\circ$, and $x = 3$. Show that the two landings are parallel.

PRACTICE AND PROBLEM SOLVING

Use the Converse of the Corresponding Angles Postulate and the given information to show that $\ell \parallel m$.

12. $\angle 3 \cong 7$
13. $m\angle 4 = 54^\circ, m\angle 8 = (7x + 5)^\circ, x = 7$
14. $m\angle 2 = (8x + 4)^\circ, m\angle 6 = (11x - 41)^\circ, x = 15$
15. $m\angle 1 = (3x + 19)^\circ, m\angle 5 = (4x + 7)^\circ, x = 12$
Use the theorems and given information to show that \( n \parallel p \).

16. \( \angle 3 \equiv \angle 6 \)

17. \( \angle 2 \equiv \angle 7 \)

18. \( m\angle 4 + m\angle 6 = 180^\circ \)

19. \( m\angle 1 = (8x - 7)^\circ, m\angle 8 = (6x + 21)^\circ, x = 14 \)

20. \( m\angle 4 = (4x + 3)^\circ, m\angle 5 = (5x - 22)^\circ, x = 25 \)

21. \( m\angle 3 = (2x + 15)^\circ, m\angle 5 = (3x + 15)^\circ, x = 30 \)

22. Complete the following two-column proof.

\[ \begin{array}{|l|l|}
\hline
\text{Statements} & \text{Reasons} \\
\hline
1. \( AB \parallel CD \) & 1. Given \\
2. \( \angle 1 \equiv \angle 3 \) & 2. a. ? \\
3. \( \angle 1 \equiv \angle 2, \angle 3 \equiv \angle 4 \) & 3. b. ? \\
4. \( \angle 2 \equiv \angle 4 \) & 4. c. ? \\
5. \( d. ? \) & 5. e. ? \\
\hline
\end{array} \]

23. **Art** Edmund Dulac used perspective when drawing the floor titles in this illustration for *The Wind’s Tale* by Hans Christian Andersen. Show that \( DJ \parallel EK \) if \( m\angle 1 = (3x + 2)^\circ, m\angle 2 = (5x - 10)^\circ \), and \( x = 6 \).

24. \( \angle 8 \equiv \angle 6 \)

25. \( \angle 7 \equiv \angle 4 \)

26. \( \angle 2 \equiv \angle 6 \)

27. \( \angle 7 \equiv \angle 5 \)

28. \( \angle 3 \equiv \angle 7 \)

29. \( m\angle 2 + m\angle 3 = 180^\circ \)

30. \( m\angle 2 = m\angle 10 \)

31. \( m\angle 8 + m\angle 9 = 180^\circ \)

32. \( \angle 1 \equiv \angle 7 \)

33. \( m\angle 10 = m\angle 6 \)

34. \( \angle 11 \equiv \angle 5 \)

35. \( m\angle 2 + m\angle 5 = 180^\circ \)

36. **Multi-Step** Two lines are intersected by a transversal so that \( \angle 1 \) and \( \angle 2 \) are corresponding angles, \( \angle 1 \) and \( \angle 3 \) are alternate exterior angles, and \( \angle 3 \) and \( \angle 4 \) are corresponding angles. If \( \angle 2 \equiv \angle 4 \), what theorem or postulate can be used to prove the lines parallel?
37. This problem will prepare you for the Multi-Step Test Prep on page 180. In the diagram, which represents the side view of a mystery spot, \( m\angle SRT = 25^\circ \), and \( m\angle SUR = 65^\circ \).
   a. Name a same-side interior angle of \( \angle SUR \) for lines \( \overline{SU} \) and \( \overline{RT} \) with transversal \( \overline{RU} \). What is its measure? Explain your reasoning.
   b. Prove that \( \overline{SU} \) and \( \overline{RT} \) are parallel.

38. Complete the flowchart proof of the Converse of the Alternate Interior Angles Theorem.
   Given: \( \angle 2 \cong \angle 3 \)
   Prove: \( \ell \parallel m \)
   Proof:
   
   ![Flowchart Proof]

39. Use the diagram to write a paragraph proof of the Converse of the Same-Side Interior Angles Theorem.
   Given: \( \angle 1 \) and \( \angle 2 \) are supplementary.
   Prove: \( \ell \parallel m \)

40. **Carpentry** A plumb bob is a weight hung at the end of a string, called a plumb line. The weight pulls the string down so that the plumb line is perfectly vertical. Suppose that the angle formed by the wall and the roof is 123° and the angle formed by the plumb line and the roof is 123°. How does this show that the wall is perfectly vertical?

41. **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for parallel lines? Explain why or why not.
   Reflexive: \( \ell \parallel \ell \)
   Symmetric: If \( \ell \parallel m \), then \( m \parallel \ell \).
   Transitive: If \( \ell \parallel m \) and \( m \parallel n \), then \( \ell \parallel n \).

42. **Write About It** Does the information given in the diagram allow you to conclude that \( a \parallel b \)? Explain.

43. Which postulate or theorem can be used to prove \( \ell \parallel m \)?
   A. Converse of the Corresponding Angles Postulate
   B. Converse of the Alternate Interior Angles Theorem
   C. Converse of the Alternate Exterior Angles Theorem
   D. Converse of the Same-Side Interior Angles Theorem
44. Two coplanar lines are cut by a transversal. Which condition does NOT guarantee that the two lines are parallel?
   A. A pair of alternate interior angles are congruent.
   B. A pair of same-side interior angles are supplementary.
   C. A pair of corresponding angles are congruent.
   D. A pair of alternate exterior angles are complementary.

45. **Gridded Response** Find the value of $x$ so that $\ell \parallel m$.

46. $\angle 1 \cong \angle 15$
47. $\angle 8 \cong \angle 14$
48. $\angle 3 \cong \angle 7$
49. $\angle 8 \cong \angle 10$
50. $\angle 6 \cong \angle 8$
51. $\angle 13 \cong \angle 11$
52. $\angle 12 + \angle 15 = 180^\circ$
53. $\angle 5 + \angle 8 = 180^\circ$
54. Write a paragraph proof that $\overline{AE} \parallel \overline{BD}$.

Use the diagram for Exercises 55 and 56.
55. **Given:** $m\angle 2 + m\angle 3 = 180^\circ$
    **Prove:** $\ell \parallel m$
56. **Given:** $m\angle 2 + m\angle 5 = 180^\circ$
    **Prove:** $\ell \parallel n$

**SPIRAL REVIEW**

Solve each equation for the indicated variable. (*Previous course*)
57. $a - b = -c$, for $a$
58. $y = \frac{1}{2}x - 10$, for $x$
59. $4y + 6x = 12$, for $y$

Write the converse, inverse, and contrapositive of each conditional statement. Find the truth value of each. (*Lesson 2-2*)
60. If an animal is a bat, then it has wings.
61. If a polygon is a triangle, then it has exactly three sides.
62. If the digit in the ones place of a whole number is 2, then the number is even.

Identify each of the following. (*Lesson 3-1*)
63. one pair of parallel segments
64. one pair of skew segments
65. one pair of perpendicular segments
Construct Parallel Lines

In Lesson 3-3, you learned one method of constructing parallel lines using a compass and straightedge. Another method, called the rhombus method, uses a property of a figure called a rhombus, which you will study in Chapter 6. The rhombus method is shown below.

**Activity 1**

1. Draw a line \( \ell \) and a point \( P \) not on the line.

   ![Diagram of line and point](image)

2. Choose a point \( Q \) on the line. Place your compass point at \( Q \) and draw an arc through \( P \) that intersects \( \ell \). Label the intersection \( R \).

   ![Diagram with arc](image)

3. Using the same compass setting as the first arc, draw two more arcs: one from \( P \), the other from \( R \). Label the intersection of the two arcs \( S \).

   ![Diagram with additional arcs](image)

4. Draw \( \overrightarrow{PS} \parallel \ell \).

   ![Final diagram with parallel line](image)

**Try This**

1. Repeat Activity 1 using a different point not on the line. Are your results the same?

2. Using the lines you constructed in Problem 1, draw transversal \( \overrightarrow{PQ} \). Verify that the lines are parallel by using a protractor to measure alternate interior angles.

3. What postulate ensures that this construction is always possible?

4. A rhombus is a quadrilateral with four congruent sides. Explain why this method is called the rhombus method.
Activity 2

1. Draw a line $\ell$ and point $P$ on a piece of patty paper.

2. Fold the paper through $P$ so that both sides of line $\ell$ match up.

3. Crease the paper to form line $m$. $P$ should be on line $m$.

4. Fold the paper again through $P$ so that both sides of line $m$ match up.

5. Crease the paper to form line $n$. Line $n$ is parallel to line $\ell$ through $P$.

Try This

5. Repeat Activity 2 using a point in a different place not on the line. Are your results the same?

6. Use a protractor to measure corresponding angles. How can you tell that the lines are parallel?

7. Draw a triangle and construct a line parallel to one side through the vertex that is not on that side.

8. Line $m$ is perpendicular to both $\ell$ and $n$. Use this statement to complete the following conjecture: If two lines in a plane are perpendicular to the same line, then ______?______.
**3-4 Perpendicular Lines**

**Objective**
Prove and apply theorems about perpendicular lines.

**Vocabulary**
perpendicular bisector distance from a point to a line

**Why learn this?**
Rip currents are strong currents that flow away from the shoreline and are perpendicular to it. A swimmer who gets caught in a rip current can get swept far out to sea. (See Example 3.)

The **perpendicular bisector** of a segment is a line perpendicular to a segment at the segment's midpoint. A construction of a perpendicular bisector is shown below.

**Construction Perpendicular Bisector of a Segment**

1. Draw \( \overline{AB} \). Open the compass wider than half of \( AB \) and draw an arc centered at \( A \).
2. Using the same compass setting, draw an arc centered at \( B \) that intersects the first arc at \( C \) and \( D \).
3. Draw \( \overline{CD} \). \( \overline{CD} \) is the perpendicular bisector of \( \overline{AB} \).

The shortest segment from a point to a line is perpendicular to the line. This fact is used to define the **distance from a point to a line** as the length of the perpendicular segment from the point to the line.

**Example 1 Distance From a Point to a Line**

A. Name the shortest segment from \( P \) to \( \overrightarrow{AC} \).
   The shortest distance from a point to a line is the length of the perpendicular segment, so \( \overline{PB} \) is the shortest segment from \( P \) to \( \overrightarrow{AC} \).

B. Write and solve an inequality for \( x \).
   \[
   PA > PB \quad \overline{PB} \text{ is the shortest segment.}
   \]
   \[
   x + 3 > 5 \quad \text{Substitute } x + 3 \text{ for } PA \text{ and } 5 \text{ for } PB.
   \]
   \[
   -3 - 3 \quad \text{Subtract } 3 \text{ from both sides of the inequality.}
   \]
   \[
   x > 2
   \]

1a. Name the shortest segment from \( A \) to \( \overrightarrow{BC} \).
1b. Write and solve an inequality for \( x \).
**Theorems**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-4-1</td>
<td>If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular. (2 intersecting lines form lin. pair of ( \equiv \Delta \rightarrow ) lines ( \perp ).)</td>
<td>( \ell \perp m )</td>
</tr>
</tbody>
</table>
| 3-4-2   | **Perpendicular Transversal Theorem**  
In a plane, if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line. | \( q \perp p \) |
| 3-4-3   | If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other. (2 lines \( \perp \) to same line \( \rightarrow \) 2 lines \( \parallel \).) | \( r \parallel s \) |

You will prove Theorems 3-4-1 and 3-4-3 in Exercises 37 and 38.

**PROOF**

**Perpendicular Transversal Theorem**

Given: \( \overrightarrow{BC} \parallel \overrightarrow{DE}, \overrightarrow{AB} \perp \overrightarrow{BC} \)

Prove: \( \overrightarrow{AB} \perp \overrightarrow{DE} \)

Proof:

It is given that \( \overrightarrow{BC} \parallel \overrightarrow{DE} \), so \( \angle ABC \equiv \angle BDE \) by the Corresponding Angles Postulate. It is also given that \( \overrightarrow{AB} \perp \overrightarrow{BC} \), so \( \angle ABC = 90^\circ \). By the definition of congruent angles, \( m\angle ABC = m\angle BDE \), so \( \angle BDE = 90^\circ \) by the Transitive Property of Equality. By the definition of perpendicular lines, \( \overrightarrow{AB} \perp \overrightarrow{DE} \).

**EXAMPLE 2**

**Proving Properties of Lines**

Write a two-column proof.

Given: \( \overrightarrow{AD} \parallel \overrightarrow{BC}, \overrightarrow{AD} \perp \overrightarrow{AB}, \overrightarrow{BC} \perp \overrightarrow{DC} \)

Prove: \( \overrightarrow{AB} \parallel \overrightarrow{DC} \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overrightarrow{AD} \parallel \overrightarrow{BC} ), ( \overrightarrow{BC} \perp \overrightarrow{DC} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overrightarrow{AD} \perp \overrightarrow{DC} )</td>
<td>2. ( \perp ) Transv. Thm.</td>
</tr>
<tr>
<td>3. ( \overrightarrow{AD} \perp \overrightarrow{AB} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overrightarrow{AB} \parallel \overrightarrow{DC} )</td>
<td>4. 2 lines ( \perp ) to same line ( \rightarrow ) 2 lines ( \parallel ).</td>
</tr>
</tbody>
</table>

2. Write a two-column proof.

Given: \( \angle EHF \equiv \angle HFG, \overrightarrow{FG} \perp \overrightarrow{GH} \)

Prove: \( \overrightarrow{EH} \perp \overrightarrow{GH} \)
**Example 3**

**Oceanography Application**

Rip currents may be caused by a sandbar parallel to the shoreline. Waves cause a buildup of water between the sandbar and the shoreline. When this water breaks through the sandbar, it flows out in a direction perpendicular to the sandbar. Why must the rip current be perpendicular to the shoreline?

The rip current forms a transversal to the shoreline and the sandbar.

The shoreline and the sandbar are parallel, and the rip current is perpendicular to the sandbar. So by the Perpendicular Transversal Theorem, the rip current is perpendicular to the shoreline.

**Check It Out!**

3. A swimmer who gets caught in a rip current should swim in a direction perpendicular to the current. Why should the path of the swimmer be parallel to the shoreline?

**Think and Discuss**

1. Describe what happens if two intersecting lines form a linear pair of congruent angles.

2. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

3. **Get Organized** Copy and complete the graphic organizer. Use the diagram and the theorems from this lesson to complete the table.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>If you are given . . .</th>
<th>Then you can conclude . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>m∠1 = m∠2</td>
<td><img src="image" alt="Table Row" /></td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>m∠2 = 90°</td>
<td><img src="image" alt="Table Row" /></td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>m∠3 = 90°</td>
<td><img src="image" alt="Table Row" /></td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>m∠2 = 90° m</td>
<td></td>
</tr>
</tbody>
</table>
### GUIDED PRACTICE

1. **Vocabulary** $\overline{CD}$ is the perpendicular bisector of $\overline{AB}$. $\overline{CD}$ intersects $\overline{AB}$ at $C$. What can you say about $\overline{AB}$ and $\overline{CD}$? What can you say about $\overline{AC}$ and $\overline{BC}$?

2. Name the shortest segment from point $E$ to $\overline{AD}$.

3. Write and solve an inequality for $x$.

4. Complete the two-column proof.

   **Given:** $\angle ABC \cong \angle CBE$, $\overline{DE} \perp \overline{AF}$

   **Prove:** $\overline{CB} \parallel \overline{DE}$

   **Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle ABC \cong \angle CBE$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{CB} \perp \overline{AF}$</td>
<td>2. a. ?</td>
</tr>
<tr>
<td>3. $\overline{CB} \parallel \overline{DE}$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ?</td>
<td>4. c. ?</td>
</tr>
</tbody>
</table>

5. **Sports** The center line in a tennis court is perpendicular to both service lines. Explain why the service lines must be parallel to each other.

### PRACTICE AND PROBLEM SOLVING

6. Name the shortest segment from point $W$ to $\overline{XZ}$.

7. Write and solve an inequality for $x$.

8. Complete the two-column proof below.

   **Given:** $\overline{AB} \perp \overline{BC}$, $m\angle 1 + m\angle 2 = 180^\circ$

   **Prove:** $\overline{BC} \perp \overline{CD}$

   **Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB} \perp \overline{BC}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle 1 + m\angle 2 = 180^\circ$</td>
<td>2. a. ?</td>
</tr>
<tr>
<td>3. $\angle 1$ and $\angle 2$ are supplementary</td>
<td>3. Def. of supplementary</td>
</tr>
<tr>
<td>4. b. ?</td>
<td>4. Converse of the Same-Side Interior Angles Theorem</td>
</tr>
<tr>
<td>5. $\overline{BC} \perp \overline{CD}$</td>
<td>5. c. ?</td>
</tr>
</tbody>
</table>
9. **Music** The frets on a guitar are all perpendicular to one of the strings. Explain why the frets must be parallel to each other.

For each diagram, write and solve an inequality for $x$.

10. \[2x - 5\]

11. \[9x - 3, 6x + 5\]

**Multi-Step** Solve to find $x$ and $y$ in each diagram.

12. \[2x, (3y - 2x)\]

13. \[6y, (5x + 4y)\]

14. \[(2x + y), (10x - 4y)\]

15. \[(x + y), 2y\]

Determine if there is enough information given in the diagram to prove each statement.

16. \[\angle 1 \cong \angle 2\]

17. \[\angle 1 \cong \angle 3\]

18. \[\angle 2 \cong \angle 3\]

19. \[\angle 2 \cong \angle 4\]

20. \[\angle 3 \cong \angle 4\]

21. \[\angle 3 \cong \angle 5\]

22. **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for perpendicular lines? Explain why or why not.

   Reflexive: \(\ell \perp \ell\)

   Symmetric: If \(\ell \perp m\), then \(m \perp \ell\).

   Transitive: If \(\ell \perp m\) and \(m \perp n\), then \(\ell \perp n\).

23. **Multi-Step Test Prep** This problem will prepare you for the Multi-Step Test Prep on page 180.

   In the diagram, which represents the side view of a mystery spot, \(QR \perp PQ\), \(PQ \parallel RS\), and \(PS \parallel QR\).

   a. Prove \(QR \perp RS\) and \(PS \perp RS\).

   b. Prove \(PQ \perp PS\).
24. **Geography** Felton Avenue, Arlee Avenue, and Viehl Avenue are all parallel. Broadway Street is perpendicular to Felton Avenue. Use the satellite photo and the given information to determine the values of \(x\) and \(y\).

25. **Estimation** Copy the diagram onto a grid with 1 cm by 1 cm squares. Estimate the distance from point \(P\) to line \(\ell\).

26. **Critical Thinking** Draw a figure to show that Theorem 3-4-3 is not true if the lines are not in the same plane.

27. Draw a figure in which \(AB\) is a perpendicular bisector of \(XY\) but \(XY\) is not a perpendicular bisector of \(AB\).

28. **Write About It** A ladder is formed by rungs that are perpendicular to the sides of the ladder. Explain why the rungs of the ladder are parallel.

29. **Construction** Construct a segment congruent to each given segment and then construct its perpendicular bisector.

30. 

31. Which inequality is correct for the given diagram?

- A 2\(x\) + 5 < 3\(x\)
- B \(x\) > 1
- C 2\(x\) + 5 > 3\(x\)
- D \(x\) > 5

32. In the diagram, \(\ell \perp m\). Find \(x\) and \(y\).

- F \(x = 5, y = 7\)
- G \(x = 7, y = 5\)
- H \(x = 90, y = 90\)
- J \(x = 10, y = 5\)

33. If \(\ell \perp m\), which statement is NOT correct?

- A \(m\angle 2 = 90^\circ\)
- B \(m\angle 1 + m\angle 2 = 180^\circ\)
- C \(\angle 1 \equiv \angle 2\)
- D \(\angle 1 \perp \angle 2\)
34. In a plane, both lines $m$ and $n$ are perpendicular to both lines $p$ and $q$. Which conclusion CANNOT be made?

A. $p \parallel q$
B. $m \parallel n$
C. $p \perp q$
D. All angles formed by lines $m$, $n$, $p$, and $q$ are congruent.

35. **Extended Response** Lines $m$ and $n$ are parallel. Line $p$ intersects line $m$ at $A$ and line $n$ at $B$, and is perpendicular to line $m$.

a. What is the relationship between line $n$ and line $p$? Draw a diagram to support your answer.

b. What is the distance from point $A$ to line $n$? What is the distance from point $B$ to line $m$? Explain.

c. How would you define the distance between two parallel lines in a plane?

**CHALLENGE AND EXTEND**

36. **Multi-Step** Find $m\angle 1$ in the diagram. *(Hint: Draw a line parallel to the given parallel lines.)*

37. Prove Theorem 3-4-1: If two intersecting lines form a linear pair of congruent angles, then the two lines are perpendicular.

38. Prove Theorem 3-4-3: If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other.

**SPIRAL REVIEW**

39. A soccer league has 6 teams. During one season, each team plays each of the other teams 2 times. What is the total number of games played in the league during one season? *(Previous course)*

Find the measure of each angle. *(Lesson 1-4)*

40. the supplement of $\angle DJE$
41. the complement of $\angle FJG$
42. the supplement of $\angle GJH$

For the given information, name the postulate or theorem that proves $\ell \parallel m$. *(Lesson 3-3)*

43. $\angle 2 \cong \angle 7$
44. $\angle 3 \cong \angle 6$
45. $m\angle 4 + m\angle 6 = 180^\circ$
3-4 Geometry Lab

Use with Lesson 3-4

Construct Perpendicular Lines

In Lesson 3-4, you learned to construct the perpendicular bisector of a segment. This is the basis of the construction of a line perpendicular to a given line through a given point. The steps in the construction are the same whether the point is on or off the line.

Activity

Copy the given line $\ell$ and point $P$.

1. Place the compass point on $P$ and draw an arc that intersects $\ell$ at two points. Label the points $A$ and $B$.

2. Construct the perpendicular bisector of $\overline{AB}$.

Try This

Copy each diagram and construct a line perpendicular to line $\ell$ through point $P$. Use a protractor to verify that the lines are perpendicular.

1.

2.

3. Follow the steps below to construct two parallel lines. Explain why $\ell \parallel n$.

Step 1 Given a line $\ell$, draw a point $P$ not on $\ell$.

Step 2 Construct line $m$ perpendicular to $\ell$ through $P$.

Step 3 Construct line $n$ perpendicular to $m$ through $P$. 
Parallel and Perpendicular Lines and Transversals

On the Spot Inside a mystery spot building, objects can appear to roll uphill, and people can look as if they are standing at impossible angles. This is because there is no view of the outside, so the room appears to be normal.

Suppose that the ground is perfectly level and the floor of the building forms a $25^\circ$ angle with the ground. The floor and ceiling are parallel, and the walls are perpendicular to the floor.

1. A table is placed in the room. The legs of the table are perpendicular to the floor, and the top is perpendicular to the legs. Draw a diagram and describe the relationship of the tabletop to the floor, walls, and ceiling of the room.

2. Find the angle of the table top relative to the ground. Suppose a ball is placed on the table. Describe what would happen and how it would appear to a person in the room.

3. Two people of the same height are standing on opposite ends of a board that makes a $25^\circ$ angle with the floor, as shown. Explain how you know that the board is parallel to the ground. What would appear to be happening from the point of view of a person inside the room?

4. In the room, a lamp hangs from the ceiling along a line perpendicular to the ground. Find the angle the line makes with the walls. Describe how it would appear to a person standing in the room.
Quiz for Lessons 3-1 Through 3-4

3-1  Lines and Angles
Identify each of the following.
1. a pair of perpendicular segments
2. a pair of skew segments
3. a pair of parallel segments
4. a pair of parallel planes

Give an example of each angle pair.
5. alternate interior angles
6. alternate exterior angles
7. corresponding angles
8. same-side interior angles

3-2  Angles Formed by Parallel Lines and Transversals
Find each angle measure.
9. \(135°\)

10. \((15x - 7)^°\)

11. \((54x + 14)^°\)

3-3  Proving Lines Parallel
Use the given information and the theorems and postulates you have learned to show that \(a \parallel b\).
12. \(m\angle 8 = (13x + 20)^°, m\angle 6 = (7x + 38)^°, x = 3\)
13. \(\angle 1 \equiv \angle 5\)
14. \(m\angle 8 + m\angle 7 = 180°\)
15. \(m\angle 8 = m\angle 4\)
16. The tower shown is supported by guy wires such that \(m\angle 1 = (3x + 12)^°, m\angle 2 = (4x - 2)^°, \) and \(x = 14\). Show that the guy wires are parallel.

3-4  Perpendicular Lines
17. Write a two-column proof.

Given: \(\angle 1 \equiv \angle 2, \ell \perp n\)
Prove: \(\ell \perp p\)
**Objectives**

Find the slope of a line.

Use slopes to identify parallel and perpendicular lines.

**Vocabulary**

rise  
run  
slope

**Why learn this?**

You can use the graph of a line to describe your rate of change, or speed, when traveling. (See Example 2.)

The slope of a line in a coordinate plane is a number that describes the steepness of the line. Any two points on a line can be used to determine the slope.

**DEFINITION**

**EXAMPLE**

The **rise** is the difference in the \( y \)-values of two points on a line.

The **run** is the difference in the \( x \)-values of two points on a line.

The **slope** of a line is the ratio of rise to run. If \((x_1, y_1)\) and \((x_2, y_2)\) are any two points on a line, the slope of the line is \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

**Slope of a Line**

<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>The <strong>rise</strong> is the difference in the ( y )-values of two points on a line.</td>
<td>![Diagram of rise and run]</td>
</tr>
<tr>
<td>The <strong>run</strong> is the difference in the ( x )-values of two points on a line.</td>
<td></td>
</tr>
<tr>
<td>The <strong>slope</strong> of a line is the ratio of rise to run.</td>
<td></td>
</tr>
</tbody>
</table>
| If \((x_1, y_1)\) and \((x_2, y_2)\) are any two points on a line, the slope of the line is \( m = \frac{y_2 - y_1}{x_2 - x_1} \). | ![

**Example 1**

**Finding the Slope of a Line**

Use the slope formula to determine the slope of each line.

**A** \( \overrightarrow{AB} \)

Substitute \((2, 3)\) for \((x_1, y_1)\) and \((7, 5)\) for \((x_2, y_2)\) in the slope formula and then simplify.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{7 - 2} = \frac{2}{5}
\]

**B** \( \overrightarrow{CD} \)

Substitute \((4, -3)\) for \((x_1, y_1)\) and \((4, 5)\) for \((x_2, y_2)\) in the slope formula and then simplify.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{4 - 4} = \frac{8}{0}
\]

The slope is undefined.

**Remember!**

A fraction with zero in the denominator is undefined because it is impossible to divide by zero.
Use the slope formula to determine the slope of each line.

Substitute \((3, 4)\) for \((x_1, y_1)\) and \((6, 4)\) for \((x_2, y_2)\) in the slope formula and then simplify.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{6 - 3} = \frac{0}{3} = 0
\]

Substitute \((6, 2)\) for \((x_1, y_1)\) and \((2, 6)\) for \((x_2, y_2)\) in the slope formula and then simplify.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{2 - 6} = \frac{4}{-4} = -1
\]

1. Use the slope formula to determine the slope of \(JK\) through \(J(3, 1)\) and \(K(2, -1)\).

**Summary: Slope of a Line**

<table>
<thead>
<tr>
<th>Positive Slope</th>
<th>Negative Slope</th>
<th>Zero Slope</th>
<th>Undefined Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph of Positive Slope" /></td>
<td><img src="image2" alt="Graph of Negative Slope" /></td>
<td><img src="image3" alt="Graph of Zero Slope" /></td>
<td><img src="image4" alt="Graph of Undefined Slope" /></td>
</tr>
</tbody>
</table>

One interpretation of slope is a *rate of change*. If \(y\) represents miles traveled and \(x\) represents time in hours, the slope gives the rate of change in miles per hour.

**Example 2**

**Transportation Application**

Tony is driving from Dallas, Texas, to Atlanta, Georgia. At 3:00 P.M., he is 180 miles from Dallas. At 5:30 P.M., he is 330 miles from Dallas. Graph the line that represents Tony’s distance from Dallas at a given time. Find and interpret the slope of the line.

Use the points \((3, 180)\) and \((5.5, 330)\) to graph the line and find the slope.

\[
m = \frac{330 - 180}{5.5 - 3} = \frac{150}{2.5} = 60
\]

The slope is 60, which means he is traveling at an average speed of 60 miles per hour.

**What if…?** Use the graph above to estimate how far Tony will have traveled by 6:30 P.M. if his average speed stays the same.
**3-5-1 Parallel Lines Theorem**
In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

**3-5-2 Perpendicular Lines Theorem**
In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$. Vertical and horizontal lines are perpendicular.

If a line has a slope of $\frac{a}{b}$, then the slope of a perpendicular line is $-\frac{b}{a}$.

The ratios $\frac{a}{b}$ and $-\frac{b}{a}$ are called **opposite reciprocals**.

### Example 3
**Determining Whether Lines Are Parallel, Perpendicular, or Neither**
Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

#### A
\( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) for \( A(2, 1), B(1, 5), C(4, 2), \) and \( D(5, -2) \)
- Slope of \( \overrightarrow{AB} \): \( \frac{5 - 1}{1 - 2} = \frac{4}{-1} = -4 \)
- Slope of \( \overrightarrow{CD} \): \( \frac{-2 - 2}{5 - 4} = \frac{-4}{1} = -4 \)

The lines have the same slope, so they are parallel.

#### B
\( \overrightarrow{ST} \) and \( \overrightarrow{UV} \) for \( S(-2, 2), T(5, -1), U(3, 4), \) and \( V(-1, -4) \)
- Slope of \( \overrightarrow{ST} \): \( \frac{-1 - 2}{5 - (-2)} = \frac{-3}{7} \)
- Slope of \( \overrightarrow{UV} \): \( \frac{-4 - 4}{-1 - 3} = \frac{-8}{-4} = 2 \)

The slopes are not the same, so the lines are not parallel. The product of the slopes is not $-1$, so the lines are not perpendicular.

#### C
\( \overrightarrow{FG} \) and \( \overrightarrow{HJ} \) for \( F(1, 1), G(2, 2), H(2, 1), \) and \( J(1, 2) \)
- Slope of \( \overrightarrow{FG} \): \( \frac{2 - 1}{2 - 1} = \frac{1}{1} = 1 \)
- Slope of \( \overrightarrow{HJ} \): \( \frac{2 - 1}{1 - 2} = \frac{1}{-1} = -1 \)

The product of the slopes is $1(-1) = -1$, so the lines are perpendicular.

#### Check It Out!
Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

3a. \( \overrightarrow{WX} \) and \( \overrightarrow{YZ} \) for \( W(3, 1), X(3, -2), Y(-2, 3), \) and \( Z(4, 3) \)
3b. \( \overrightarrow{KL} \) and \( \overrightarrow{MN} \) for \( K(-4, 4), L(-2, -3), M(3, 1), \) and \( N(-5, -1) \)
3c. \( \overrightarrow{BC} \) and \( \overrightarrow{DE} \) for \( B(1, 1), C(3, 5), D(-2, -6), \) and \( E(3, 4) \)
GUIDED PRACTICE

1. **Vocabulary** The slope of a line is the ratio of its ___ to its ___. (rise or run)

Use the slope formula to determine the slope of each line.

2. $\overrightarrow{MN}$

3. $\overrightarrow{CD}$

4. $\overrightarrow{AB}$

5. $\overrightarrow{ST}$

6. **Biology** A migrating bird flying at a constant speed travels 80 miles by 8:00 A.M. and 200 miles by 11:00 A.M. Graph the line that represents the bird’s distance traveled. Find and interpret the slope of the line.

Graph each pair of lines. Use slopes to determine whether the lines are parallel, perpendicular, or neither.

7. $\overrightarrow{HJ}$ and $\overrightarrow{KM}$ for $H(3, 2), J(4, 1), K(-2, -4),$ and $M(-1, -5)$

8. $\overrightarrow{LM}$ and $\overrightarrow{NP}$ for $L(-2, 2), M(2, 5), N(0, 2),$ and $P(3, -2)$

9. $\overrightarrow{QR}$ and $\overrightarrow{ST}$ for $Q(6, 1), R(-2, 4), S(5, 3),$ and $T(-3, -1)$
PRACTICE AND PROBLEM SOLVING

Use the slope formula to determine the slope of each line.

10. \( \overrightarrow{AB} \)

11. \( \overrightarrow{CD} \)

12. \( \overrightarrow{EF} \)

13. \( \overrightarrow{GH} \)

14. Aviation A pilot traveling at a constant speed flies 100 miles by 2:30 P.M. and 475 miles by 5:00 P.M. Graph the line that represents the pilot’s distance flown. Find and interpret the slope of the line.

15. \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) for \( A(2, -1), B(7, 2), C(2, -3), \) and \( D(-3, -6) \)

16. \( \overrightarrow{XY} \) and \( \overrightarrow{ZW} \) for \( X(-2, 5), Y(6, -2), Z(-3, 6), \) and \( W(4, 0) \)

17. \( \overrightarrow{JK} \) and \( \overrightarrow{JL} \) for \( J(-4, -2), K(4, -2), \) and \( L(-4, 6) \)

18. Geography A point on a river has an elevation of about 1150 meters above sea level. The length of the river from that point to where it enters the sea is about 2400 km. Find and interpret the slope of the river.

For \( F(7, 6), G(-3, 5), H(-2, -3), J(4, -2), \) and \( K(6, 1) \), find each slope.

19. \( \overrightarrow{FG} \)

20. \( \overrightarrow{GJ} \)

21. \( \overrightarrow{HK} \)

22. \( \overrightarrow{GK} \)

23. Critical Thinking The slope of \( \overrightarrow{AB} \) is greater than 0 and less than 1. Write an inequality for the slope of a line perpendicular to \( \overrightarrow{AB} \).

24. Write About It Two cars are driving at the same speed. What is true about the lines that represent the distance traveled by each car at a given time?

25. This problem will prepare you for the Multi-Step Test Prep on page 200.

A traffic engineer calculates the speed of vehicles as they pass a traffic light. While the light is green, a taxi passes at a constant speed. After 2 s the taxi is 132 ft past the light. After 5 s it is 330 ft past the light.

a. Find the speed of the taxi in feet per second.

b. Use the fact that 22 ft/s = 15 mi/h to find the taxi’s speed in miles per hour.
26. \( AB \perp CD \) for \( A(1, 3), B(4, -2), C(6, 1), \) and \( D(x, y) \). Which are possible values of \( x \) and \( y \)?
   - \( \text{A} \) \( x = 1, y = -2 \)
   - \( \text{B} \) \( x = 3, y = 6 \)
   - \( \text{C} \) \( x = 3, y = -4 \)
   - \( \text{D} \) \( x = -2, y = -4 \)

27. Classify \( \overrightarrow{MN} \) and \( \overrightarrow{PQ} \) for \( M(-3, 1), N(1, 3), P(8, 4), \) and \( Q(2, 1) \).
   - \( \text{F} \) Parallel
   - \( \text{G} \) Perpendicular
   - \( \text{H} \) Vertical
   - \( \text{I} \) Skew

28. In the formula \( d = rt \), \( d \) represents distance, and \( r \) represents the rate of change, or slope. Which ray on the graph represents a slope of 45 miles per hour?
   - \( \text{A} \) A
   - \( \text{B} \) B
   - \( \text{C} \) C
   - \( \text{D} \) D

CHALLENGE AND EXTEND
Use the given information to classify \( \overrightarrow{JK} \) for \( J(a, b) \) and \( K(c, d) \).
29. \( a = c \)
30. \( b = d \)
31. The vertices of square \( ABCD \) are \( A(0, -2), B(6, 4), C(0, 10), D(-6, 4) \).
   a. Show that the opposite sides are parallel.
   b. Show that the consecutive sides are perpendicular.
   c. Show that all sides are congruent.
32. \( \overrightarrow{ST} \parallel \overrightarrow{WV} \) for \( S(-3, 5), T(1, -1), V(x, -3), \) and \( W(1, y) \). Find a set of possible values for \( x \) and \( y \).
33. \( \overrightarrow{MN} \perp \overrightarrow{PQ} \) for \( M(2, 1), N(-3, 0), P(x, 4), \) and \( Q(3, y) \). Find a set of possible values for \( x \) and \( y \).

SPIRAL REVIEW
Find the \( x \)- and \( y \)-intercepts of the line that contains each pair of points.
(Previous course)
34. \((-5, 0) \) and \((0, -5) \)
35. \((0, 1) \) and \((2, -7) \)
36. \((1, -3) \) and \((3, 3) \)

Use the given paragraph proof to write a two-column proof. (Lesson 2-7)
37. Given: \( \angle 1 \) is supplementary to \( \angle 3 \).
   Prove: \( \angle 2 \equiv \angle 3 \)
   Proof: It is given that \( \angle 1 \) is supplementary to \( \angle 3 \). \( \angle 1 \) and \( \angle 2 \) are a linear pair by the definition of a linear pair. By the Linear Pair Theorem, \( \angle 1 \) and \( \angle 2 \) are supplementary. Thus \( \angle 2 \equiv \angle 3 \) by the Congruent Supplements Theorem.

Given that \( m\angle 2 = 75^\circ \), tell whether each statement is true or false. Justify your answer with a postulate or theorem. (Lesson 3-2)
38. \( \angle 1 \equiv \angle 8 \)
39. \( \angle 2 \equiv \angle 6 \)
40. \( \angle 3 \equiv \angle 5 \)
Explore Parallel and Perpendicular Lines

A graphing calculator can help you explore graphs of parallel and perpendicular lines. To graph a line on a calculator, you can enter the equation of the line in slope-intercept form. The slope-intercept form of the equation of a line is \( y = mx + b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept. For example, the line \( y = 2x + 3 \) has a slope of 2 and crosses the \( y \)-axis at (0, 3).

Activity 1

1. On a graphing calculator, graph the lines \( y = 3x - 4 \), \( y = -3x - 4 \), and \( y = 3x + 1 \). Which lines appear to be parallel? What do you notice about the slopes of the parallel lines?

2. Graph \( y = 2x \). Experiment with other equations to find a line that appears parallel to \( y = 2x \). If necessary, graph \( y = 2x \) on graph paper and construct a parallel line. What is the slope of this new line?

3. Graph \( y = -\frac{1}{2}x + 3 \). Try to graph a line that appears parallel to \( y = -\frac{1}{2}x + 3 \). What is the slope of this new line?

Try This

1. Create two new equations of lines that you think will be parallel. Graph these to confirm your conjecture.

2. Graph two lines that you think are parallel. Change the window settings on the calculator. Do the lines still appear parallel? Describe your results.

3. Try changing the \( y \)-intercepts of one of the parallel lines. Does this change whether the lines appear to be parallel?
On a graphing calculator, perpendicular lines may not appear to be perpendicular on the screen. This is because the unit distances on the x-axis and y-axis can have different lengths. To make sure that the lines appear perpendicular on the screen, use a square window, which shows the x-axis and y-axis as having equal unit distances.

One way to get a square window is to use the Zoom feature. On the Zoom menu, the ZDecimal and ZSquare commands change the window to a square window. The ZStandard command does not produce a square window.

**Activity 2**

1. Graph the lines $y = x$ and $y = -x$ in a square window. Do the lines appear to be perpendicular?

2. Graph $y = 3x - 2$ in a square window. Experiment with other equations to find a line that appears perpendicular to $y = 3x - 2$. If necessary, graph $y = 3x - 2$ on graph paper and construct a perpendicular line. What is the slope of this new line?

3. Graph $y = \frac{2}{3}x$ in a square window. Try to graph a line that appears perpendicular to $y = \frac{2}{3}x$. What is the slope of this new line?

**Try This**

4. Create two new equations of lines that you think will be perpendicular. Graph these in a square window to confirm your conjecture.

5. Graph two lines that you think are perpendicular. Change the window settings on the calculator. Do the lines still appear perpendicular? Describe your results.

6. Try changing the y-intercepts of one of the perpendicular lines. Does this change whether the lines appear to be perpendicular?
Objective
Graph lines and write their equations in slope-intercept and point-slope form.
Classify lines as parallel, intersecting, or coinciding.

Vocabulary
point-slope form
slope-intercept form

Why learn this?
The cost of some health club plans includes a one-time enrollment fee and a monthly fee. You can use the equations of lines to determine which plan is best for you. (See Example 4.)

The equation of a line can be written in many different forms. The point-slope and slope-intercept forms of a line are equivalent. Because the slope of a vertical line is undefined, these forms cannot be used to write the equation of a vertical line.

Forms of the Equation of a Line

<table>
<thead>
<tr>
<th>FORM</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>The point-slope form of a line is $y - y_1 = m(x - x_1)$, where $m$ is the slope and $(x_1, y_1)$ is a given point on the line.</td>
<td>$y - 3 = 2(x - 4)$, $m = 2$, $(x_1, y_1) = (3, 4)$</td>
</tr>
<tr>
<td>The slope-intercept form of a line is $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept.</td>
<td>$y = 3x + 6$, $m = 3$, $b = 6$</td>
</tr>
<tr>
<td>The equation of a vertical line is $x = a$, where $a$ is the $x$-intercept.</td>
<td>$x = 5$</td>
</tr>
<tr>
<td>The equation of a horizontal line is $y = b$, where $b$ is the $y$-intercept.</td>
<td>$y = 2$</td>
</tr>
</tbody>
</table>

You will use a proof to derive the slope-intercept form of a line in Exercise 54.

Point-Slope Form of a Line

Given: The slope of a line through points $(x_1, y_1)$ and $(x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
Prove: The equation of the line through $(x_1, y_1)$ with slope $m$ is $y - y_1 = m(x - x_1)$.

Proof:
Let $(x, y)$ be any point on the line.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}
\]

\[
m = \frac{y - y_1}{x - x_1} \quad \text{Substitute $(x, y)$ for $(x_2, y_2)$}.
\]

\[
(x - x_1)m = (x - x_1)\frac{y - y_1}{x - x_1} \quad \text{Multiply both sides by $(x - x_1)$}.
\]

\[
m(x - x_1) = (y - y_1) \quad \text{Simplify}.
\]

\[
y - y_1 = m(x - x_1) \quad \text{Sym. Prop. of =}
\]
**Writing Equations of Lines**

Write the equation of each line in the given form.

**A** the line with slope 3 through (2, 1) in point-slope form

\[ y - y_1 = m(x - x_1) \quad \text{Point-slope form} \]
\[ y - 1 = 3(x - 2) \quad \text{Substitute 3 for } m, 2 \text{ for } x_1, \text{ and } 1 \text{ for } y_1. \]

**B** the line through (0, 4) and (−1, 2) in slope-intercept form

\[ m = \frac{2 - 4}{-1 - 0} = \frac{-2}{-1} = 2 \quad \text{Find the slope.} \]
\[ y = mx + b \quad \text{Slope-intercept form} \]
\[ 4 = 2(0) + b \]
\[ 4 = b \quad \text{Substitute 2 for } m, 0 \text{ for } x, \text{ and 4 for } y \text{ to find } b. \]
\[ y = 2x + 4 \quad \text{Write in slope-intercept form using } m = 2 \text{ and } b = 4. \]

**C** the line with x-intercept 2 and y-intercept 3 in point-slope form

\[ m = \frac{3 - 0}{0 - 2} = -\frac{3}{2} \quad \text{Use the points } (2, 0) \text{ and } (0, 3) \text{ to find the slope.} \]
\[ y - y_1 = m(x - x_1) \quad \text{Point-slope form} \]
\[ y - 0 = -\frac{3}{2}(x - 2) \quad \text{Substitute } -\frac{3}{2} \text{ for } m, 2 \text{ for } x_1, \text{ and } 0 \text{ for } y_1. \]
\[ y = -\frac{3}{2}(x - 2) \quad \text{Simplify.} \]

Write the equation of each line in the given form.

1a. the line with slope 0 through (4, 6) in slope-intercept form

1b. the line through (−3, 2) and (1, 2) in point-slope form

**Graphing Lines**

Graph each line.

**A** \[ y = \frac{3}{2}x + 3 \]

The equation is given in slope-intercept form, with a slope of \( \frac{3}{2} \) and a y-intercept of 3.
Plot the point (0, 3) and then rise 3 and run 2 to find another point.
Draw the line containing the two points.

**B** \[ y + 3 = -2(x - 1) \]

The equation is given in point-slope form, with a slope of \( -2 = -\frac{2}{1} \) through the point \((1, -3)\).
Plot the point \((1, -3)\) and then rise −2 and run 1 to find another point.
Draw the line containing the two points.
Graph the line.

**C**

\[ x = 3 \]

The equation is given in the form for a vertical line with an \( x \)-intercept of 3. The equation tells you that the \( x \)-coordinate of every point on the line is 3. Draw the vertical line through \((3, 0)\).

Graph each line.

**2a.** \( y = 2x - 3 \)

**2b.** \( y - 1 = -\frac{2}{3}(x + 2) \)

**2c.** \( y = -4 \)

A system of two linear equations in two variables represents two lines. The lines can be parallel, intersecting, or coinciding. Lines that coincide are the same line, but the equations may be written in different forms.

### Pairs of Lines

<table>
<thead>
<tr>
<th>Parallel Lines</th>
<th>Intersecting Lines</th>
<th>Coinciding Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 5x + 8 )</td>
<td>( y = 2x - 5 )</td>
<td>( y = 2x - 4 )</td>
</tr>
<tr>
<td>( y = 5x - 4 )</td>
<td>( y = 4x + 3 )</td>
<td>( y = 2x - 4 )</td>
</tr>
<tr>
<td>Same slope</td>
<td>Different slopes</td>
<td>Same slope</td>
</tr>
<tr>
<td>different ( y )-intercept</td>
<td></td>
<td>same ( y )-intercept</td>
</tr>
</tbody>
</table>

### Example 3

**Classifying Pairs of Lines**

Determine whether the lines are parallel, intersect, or coincide.

**A**

\( y = 2x + 3, y = 2x - 1 \)

Both lines have a slope of 2, and the \( y \)-intercepts are different. So the lines are parallel.

**B**

\( y = 3x - 5, 6x - 2y = 10 \)

Solve the second equation for \( y \) to find the slope-intercept form.

\[
6x - 2y = 10 \\
-2y = -6x + 10 \\
y = 3x - 5
\]

Both lines have a slope of 3 and a \( y \)-intercept of \(-5\), so they coincide.

**C**

\( 3x + 2y = 7, 3y = 4x + 7 \)

Solve both equations for \( y \) to find the slope-intercept form.

\[
3x + 2y = 7 \\
2y = -3x + 7 \\
y = -\frac{3}{2}x + \frac{7}{2}
\]

\[
3y = 4x + 7 \\
y = \frac{4}{3}x + \frac{7}{3}
\]

The slope is \( \frac{4}{3} \) and \( \frac{-3}{2} \). The lines have different slopes, so they intersect.

**3.** Determine whether the lines \( 3x + 5y = 2 \) and \( 3x + 6 = -5y \) are parallel, intersect, or coincide.
Audrey is trying to decide between two health club plans. After how many months would both plans’ total costs be the same?

**Understand the Problem**

The answer is the number of months after which the costs of the two plans would be the same. Plan A costs $140 for enrollment and $35 per month. Plan B costs $60 for enrollment and $55 per month.

**Make a Plan**

Write an equation for each plan, and then graph the equations. The solution is the intersection of the two lines. Find the intersection by solving the system of equations.

**Solve**

Plan A: \[ y = 35x + 140 \]
Plan B: \[ y = 55x + 60 \]

Subtract the second equation from the first.

\[ 0 = -20x + 80 \]

Solve for \( x \).

\[ x = 4 \]

Substitute 4 for \( x \) in the first equation.

\[ y = 35(4) + 140 = 280 \]

The lines cross at (4, 280). Both plans cost $280 after 4 months.

**Look Back**

Check your answer for each plan in the original problem. For 4 months, plan A costs $140 plus $35(4) = $140 + $140 = $280. Plan B costs $60 + $55(4) = $60 + $220 = $280, so the plans cost the same.

Use the information above to answer the following.

4. **What if...?** Suppose the rate for Plan B was also $35 per month. What would be true about the lines that represent the cost of each plan?

**THINK AND DISCUSS**

1. Explain how to use the slopes and \( y \)-intercepts to determine if two lines are parallel.

2. Describe the relationship between the slopes of perpendicular lines.

3. **GET ORGANIZED** Copy and complete the graphic organizer.
GUIDED PRACTICE

1. **Vocabulary** How can you recognize the slope-intercept form of an equation?

Write the equation of each line in the given form.

2. the line through (4, 7) and (−2, 1) in slope-intercept form

3. the line through (−4, 2) with slope \( \frac{3}{4} \) in point-slope form.

4. the line with \( x \)-intercept 4 and \( y \)-intercept −2 in slope-intercept form

Graph each line.

5. \( y = -3x + 4 \)

6. \( y + 4 = \frac{2}{3}(x - 6) \)

7. \( x = 5 \)

Determine whether the lines are parallel, intersect, or coincide.

8. \( y = -3x + 4, y = -3x + 1 \)

9. \( 6x - 12y = -24, 3y = 2x + 18 \)

10. \( y = \frac{1}{3}x + \frac{2}{3}, 3y = x + 2 \)

11. \( 4x + 2y = 10, y = -2x + 15 \)

12. **Transportation** A speeding ticket in Conroe costs $115 for the first 10 mi/h over the speed limit and $1 for each additional mi/h. In Lakeville, a ticket costs $50 for the first 10 mi/h over the speed limit and $10 for each additional mi/h. If the speed limit is 55 mi/h, at what speed will the tickets cost approximately the same?

PRACTICE AND PROBLEM SOLVING

Write the equation of each line in the given form.

13. the line through (0, −2) and (4, 6) in point-slope form

14. the line through (5, 2) and (−2, 2) in slope-intercept form

15. the line through (6, −4) with slope \( \frac{2}{3} \) in point-slope form

Graph each line.

16. \( y - 7 = x + 4 \)

17. \( y = \frac{1}{2}x - 2 \)

18. \( y = 2 \)

Determine whether the lines are parallel, intersect, or coincide.

19. \( y = x - 7, y = -x + 3 \)

20. \( y = \frac{5}{2}x + 4, 2y = 5x - 4 \)

21. \( x + 2y = 6, y = -\frac{1}{2}x + 3 \)

22. \( 7x + 2y = 10, 3y = 4x - 5 \)

23. **Business** Chris is comparing two sales positions that he has been offered. The first pays a weekly salary of $375 plus a 20% commission. The second pays a weekly salary of $325 plus a 25% commission. How much must he make in sales per week for the two jobs to pay the same?

Write the equation of each line in slope-intercept form. Then graph the line.

24. through (−6, 2) and (3, 6)

25. horizontal line through (2, 3)

26. through (5, −2) with slope \( \frac{2}{3} \)

27. \( x \)-intercept 4, \( y \)-intercept −3

Write the equation of each line in point-slope form. Then graph the line.

28. slope \( -\frac{1}{2} \), \( y \)-intercept 2

29. slope \( \frac{3}{4} \), \( x \)-intercept −2

30. through (5, −1) with slope −1

31. through (4, 6) and (−2, −5)
32. ***ERROR ANALYSIS*** Write the equation of the line with slope $-2$ through the point $(-4, 3)$ in slope-intercept form. Which equation is incorrect? Explain.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 3 - 2(x + 4)$</td>
<td>$y + 4 = -2(x - 3)$</td>
</tr>
<tr>
<td>$y = 2x - 8$</td>
<td>$y + 4 = -2x + 6$</td>
</tr>
<tr>
<td>$y = -2x - 5$</td>
<td>$y = -2x + 2$</td>
</tr>
</tbody>
</table>

Determine whether the lines are perpendicular.

33. $y = 3x - 5, y = -3x + 1$

34. $y = -x + 1, y = x + 2$

35. $y = -\frac{2}{3}x + 5, y = \frac{3}{2}x - 8,$

36. $y = -2x + 4, y = -\frac{1}{2}x - 2$

**Multi-Step** Given the equation of the line and point $P$ not on the line, find the equation of a line parallel to the given line and a line perpendicular to the given line through the given point.

37. $y = 3x + 7, P(2, 3)$

38. $y = -2x - 5, P(-1, 4)$

39. $4x + 3y = 8, P(4, -2)$

40. $2x - 5y = 7, P(-2, 4)$

**Multi-Step** Use slope to determine if each triangle is a right triangle. If so, which angle is the right angle?

41. $A(-5, 3), B(0, -2), C(5, 3)$

42. $D(1, 0), E(2, 7), F(5, 1)$

43. $G(3, 4), H(-3, 4), J(1, -2)$

44. $K(-2, 4), L(2, 1), M(1, 8)$

45. **Food** A restaurant charges $8 for a large cheese pizza plus $1.50 per topping. Another restaurant charges $11 for a large cheese pizza plus $0.75 per topping. How many toppings does a pizza have that costs the same at both restaurants?

46. **Estimation** Estimate the solution of the system of equations represented by the lines in the graph.

Write the equation of the perpendicular bisector of the segment with the given endpoints.

47. $(2, 5)$ and $(4, 9)$

48. $(1, 1)$ and $(3, 1)$

49. $(1, 3)$ and $(-1, 4)$

50. $(-3, 2)$ and $(-3, -10)$

51. Line $\ell$ has equation $y = -\frac{1}{2}x + 4$, and point $P$ has coordinates $(3, 5)$.
   a. Find the equation of line $m$ that passes through $P$ and is perpendicular to $\ell$.
   b. Find the coordinates of the intersection of $\ell$ and $m$.
   c. What is the distance from $P$ to $\ell$?

52. Line $p$ has equation $y = x + 3$, and line $q$ has equation $y = x - 1$.
   a. Find the equation of a line $r$ that is perpendicular to $p$ and $q$.
   b. Find the coordinates of the intersection of $p$ and $r$ and the coordinates of the intersection of $q$ and $r$.
   c. Find the distance between lines $p$ and $q$. 
53. This problem will prepare you for the Multi-Step Test Prep on page 200. For a car moving at 60 mi/h, the equation \( d = 88t \) gives the distance in feet \( d \) that the car travels in \( t \) seconds.
   a. Graph the line \( d = 88t \).
   b. On the same graph you made for part a, graph the line \( d = 300 \). What does the intersection of the two lines represent?
   c. Use the graph to estimate the number of seconds it takes the car to travel 300 ft.

54. Prove the slope-intercept form of a line, given the point-slope form.
   **Given:** The equation of the line through \((x_1, y_1)\) with slope \(m\) is \(y - y_1 = m(x - x_1)\).
   **Prove:** The equation of the line through \((0, b)\) with slope \(m\) is \(y = mx + b\).
   **Plan:** Substitute \((0, b)\) for \((x_1, y_1)\) in the equation \(y - y_1 = m(x - x_1)\) and simplify.

55. **Data Collection** Use a graphing calculator and a motion detector to do the following: Walk in front of the motion detector at a constant speed, and write the equation of the resulting graph.

56. **Critical Thinking** A line contains the points \((-4, 6)\) and \((2, 2)\). Write a convincing argument that the line crosses the \(x\)-axis at \((5, 0)\). Include a graph to verify your argument.

57. **Write About It** Determine whether the lines are parallel. Use slope to explain your answer.

58. Which graph best represents a solution to this system of equations?
   \[
   \begin{align*}
   -3x + y &= 7 \\
   2x + y &= -3
   \end{align*}
   \]
59. Which line is parallel to the line with the equation \( y = -2x + 5 \)?
   - F \( \overrightarrow{AB} \) through \( A(2, 3) \) and \( B(1, 1) \)
   - H \( 4x + 2y = 10 \)
   - G \( y = -\frac{1}{2}x - 3 \)
   - J \( x + \frac{1}{2}y = 1 \)

60. Which equation best describes the graph shown?

   A \( y = \frac{3}{2}x + 3 \)
   - B \( y = 3x - \frac{2}{3} \)
   - C \( y = \frac{2}{3}x + 2 \)
   - D \( y = -\frac{2}{3}x + 3 \)

61. Which line includes the points \((-4, 2)\) and \((6, -3)\)?
   - F \( y = 2x - 4 \)
   - G \( y = 2x \)
   - H \( y = -\frac{1}{2}x - 4 \)
   - J \( y = -\frac{1}{2}x \)

**CHALLENGE AND EXTEND**

62. A right triangle is formed by the \(x\)-axis, the \(y\)-axis, and the line \( y = -2x + 5 \). Find the length of the hypotenuse.

63. If the length of the hypotenuse of a right triangle is 17 units and the legs lie along the \(x\)-axis and \(y\)-axis, find a possible equation that describes the line that contains the hypotenuse.

64. Find the equations of three lines that form a triangle with a hypotenuse of 13 units.

65. **Multi-Step** Are the points \((-2, -4)\), \((5, -2)\) and \((2, -3)\) collinear? Explain the method you used to determine your answer.

66. For the line \( y = x + 1 \) and the point \( P(3, 2) \), let \( d \) represent the distance from \( P \) to a point \((x, y)\) on the line.
   - a. Write an expression for \( d^2 \) in terms of \( x \) and \( y \). Substitute the expression \( x + 1 \) for \( y \) and simplify.
   - b. How could you use this expression to find the shortest distance from \( P \) to the line? Compare your result to the distance along a perpendicular line.

**SPIRAL REVIEW**

67. The cost of renting DVDs from an online company is $5.00 per month plus $2.50 for each DVD rented. Write an equation for the total cost \( c \) of renting \( d \) DVDs from the company in one month. Graph the equation. How many DVDs did Sean rent from the company if his total bill for one month was $20.00? *(Previous course)*

Use the coordinate plane for Exercises 68–70.

Find the coordinates of the midpoint of each segment.

(Lesson 1-6)

68. \( \overrightarrow{AB} \)
69. \( \overrightarrow{BC} \)
70. \( \overrightarrow{AC} \)

Use the slope formula to find the slope of each segment.

(Lesson 3-5)

71. \( \overrightarrow{AB} \)
72. \( \overrightarrow{BC} \)
73. \( \overrightarrow{AC} \)
Example 1

The table shows several possible measures of an angle and its supplement. Graph the points in the table. Then draw the line that best represents the data and write the equation of the line.

Step 1
Use the table to write ordered pairs \((x, 180 - x)\) and then plot the points.

\[(30, 150), (60, 120), (90, 90), (120, 60), (150, 30)\]

Step 2
Draw a line that passes through all the points.

<table>
<thead>
<tr>
<th>x</th>
<th>(y = 180 - x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>120</td>
<td>60</td>
</tr>
<tr>
<td>150</td>
<td>30</td>
</tr>
</tbody>
</table>

Step 3
Choose two points from the line, such as \((30, 150)\) and \((120, 60)\). Use them to find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 150}{120 - 30} = \frac{-90}{90} = -1
\]

Step 4
Use the point-slope form to find the equation of the line and then simplify.

\[
y - y_1 = m(x - x_1) \quad \text{Point-slope form}
\]

\[
y - 150 = -1(x - 30) \quad \text{Substitute} \ (30, 150) \text{ for} \ (x_1, y_1) \text{ and} \ -1 \text{ for} \ m.
\]

\[
y = -x + 180 \quad \text{Simplify.}
\]
If you can draw a line through all the points in a set of data, the relationship is linear. If the points are close to a line, you can approximate the relationship with a line of best fit.

**Example 2**

A physical therapist evaluates a client’s progress by measuring the angle of motion of an injured joint. The table shows the angle of motion of a client’s wrist over six weeks. Estimate the equation of the line of best fit.

<table>
<thead>
<tr>
<th>Week</th>
<th>Angle Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>46</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>54</td>
</tr>
<tr>
<td>6</td>
<td>62</td>
</tr>
</tbody>
</table>

**Step 1**

Use the table to write ordered pairs and then plot the points.

\[(1, 30), (2, 36), (3, 46), (4, 48), (5, 54), (6, 62)\]

**Step 2**

Use a ruler to estimate a line of best fit. Try to get the edge of the ruler closest to all the points on the line.

**Step 3**

A line passing through \( (2, 36) \) and \( (6, 62) \) seems to be closest to all the points. Draw this line. Use the points \( (2, 36) \) and \( (6, 62) \) to find the slope of the line.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{62 - 36}{6 - 2} = 6.5
\]

**Step 4**

Use the point-slope form to find the equation of the line and then simplify.

\[
y - y_1 = m(x - x_1)
\]

Point-slope form

\[
y - 36 = 6.5(x - 2)
\]

Substitute \( (2, 36) \) for \( (x_1, y_1) \) and 6.5 for \( m \).

\[
y = 6.5x + 23
\]

Simplify.

---

**Try This**

Estimate the equation of the line of best fit for each relationship.

1. ![Graph](image1)

2. the relationship between an angle and its complement

3. **Data Collection** Use a graphing calculator and a motion detector to do the following: Set the equipment so that the graph shows distance on the \( y \)-axis and time on the \( x \)-axis. Walk in front of the motion detector while varying your speed slightly and use the resulting graph.
Coordinate Geometry

Red Light, Green Light  When a driver approaches an intersection and sees a yellow traffic light, she must decide if she can make it through the intersection before the light turns red. Traffic engineers use graphs and equations to study this situation.

1. Traffic engineers can set the duration of the yellow lights on Lincoln Road for any length of time $t$ up to 10 seconds. For each value of $t$, there is a critical distance $d$. If a car moving at the speed limit is more than $d$ feet from the light when it turns yellow, the driver will have to stop. If the car is less than $d$ feet from the light, the driver can continue through the intersection. The graph shows the relationship between $t$ and $d$. Find the speed limit on Lincoln Road in miles per hour. (Hint: 22 ft/s = 15 mi/h)

2. Traffic engineers use the equation $d = \frac{22}{15} st$ to determine the critical distance for various durations of a yellow light. In the equation, $s$ is the speed limit. The speed limit on Porter Street is 45 mi/h. Write the equation of the critical distance for a yellow light on Porter Street and then graph the line. Does this line intersect the line for Lincoln Road? If so, where? Is the line for Porter Street steeper or flatter than the line for Lincoln Road? Explain how you know.
Quiz for Lesson 3-5 Through 3-6

3-5  Slopes of Lines

Use the slope formula to determine the slope of each line.
1. \( \frac{AC}{CD} \)  2. \( \frac{CD}{AB} \)  3. \( \frac{AB}{BD} \)

Find the slope of the line through the given points.
5. \( M(2, 3) \) and \( N(0, 7) \)  6. \( F(-1, 4) \) and \( G(5, -1) \)
7. \( P(4, 0) \) and \( Q(1, -3) \)  8. \( K(4, 2) \) and \( L(-3, 2) \)

9. Sonia is walking 2.5 miles home from school. She leaves at 4:00 P.M., and gets home at 4:45 P.M. Graph the line that represents Sonia’s distance from school at a given time.

Find and interpret the slope of the line.

Graph each pair of lines and use their slopes to determine if they are parallel, perpendicular, or neither.
10. \( \overrightarrow{EF} \) and \( \overrightarrow{GH} \) for \( E(-2, 3) \), \( F(6, 1) \), \( G(6, 4) \), and \( H(2, 5) \)
11. \( \overrightarrow{JK} \) and \( \overrightarrow{LM} \) for \( J(4, 3) \), \( K(5, -1) \), \( L(-2, 4) \), and \( M(3, -5) \)
12. \( \overrightarrow{NP} \) and \( \overrightarrow{QR} \) for \( N(5, -3) \), \( P(0, 4) \), \( Q(-3, -2) \), and \( R(4, 3) \)

3-6  Lines in the Coordinate Plane

Write the equation of each line in the given form.
14. the line through \((3, 8)\) and \((-3, 4)\) in slope-intercept form
15. the line through \((-5, 4)\) with slope \( \frac{2}{3} \) in point-slope form
16. the line with \( y \)-intercept 2 through the point \((4, 1)\) in slope-intercept form

Graph each line.
17. \( y = -2x + 5 \)  18. \( y + 3 = \frac{1}{4}(x - 4) \)  19. \( x = 3 \)

Write the equation of each line.
20. \( y = -2x + 5 \)  21. \( 3x + 2y = 8 \)  22. \( y = -\frac{3}{2}x + 4 \)

Determine whether the lines are parallel, intersect, or coincide.
23. \( y = -2x + 5 \)
\( y = -2x - 5 \)
24. \( 3x + 2y = 8 \)
\( y = -\frac{3}{2}x + 4 \)
25. \( y = 4x - 5 \)
\( 3x + 4y = 7 \)
**Vocabulary**

- alternate exterior angles . . . . 147
- alternate interior angles . . . . 147
- corresponding angles . . . . . 147
- distance from a point to a line . . . . . . . . . 172
- parallel lines . . . . . . . . . . . . . . 146
- parallel planes . . . . . . . . . . . . . . 146
- perpendicular bisector . . . . . 172
- perpendicular lines . . . . . 146
- point-slope form . . . . . . . . . . . . . . 190
- rise . . . . . . . . . . . . . . 182
- run . . . . . . . . . . . . . . 182
- same-side interior angles . . . . 147
- skew lines . . . . . . . . . . . . . . 146
- slope . . . . . . . . . . . . . . 182
- slope-intercept form . . . . . 190
- transversal . . . . . . . . . . . . . . 147
- alternate exterior angles
- alternate interior angles
- corresponding angles
- distance from a point to a line
- parallel lines
- parallel planes
- perpendicular bisector
- perpendicular lines
- point-slope form
- rise
- run
- same-side interior angles
- skew lines
- slope
- slope-intercept form
- transversal

Complete the sentences below with vocabulary words from the list above.

1. Angles on opposite sides of a transversal and between the lines it intersects are _____.
2. Lines that are in different planes are _____.
3. A(n) ____ is a line that intersects two coplanar lines at two points.
4. The ____ is used to write the equation of a line with a given slope that passes through a given point.
5. The slope of a line is the ratio of the ____ to the ____.

3-1 **Lines and Angles** *(pp. 146–151)*

**Examples**

Identify each of the following.

- a pair of parallel segments
  \[ \overline{AB} \parallel \overline{CD} \]
- a pair of parallel planes
  \[ \text{plane } \overline{ABC} \parallel \text{plane } \overline{EFG} \]
- a pair of perpendicular segments
  \[ \overline{AB} \perp \overline{AE} \]
- a pair of skew segments
  \[ \overline{AB} \text{ and } \overline{FG} \text{ are skew.} \]

**Exercises**

Identify each of the following.

- a pair of skew segments
- a pair of parallel segments
- a pair of perpendicular segments
- a pair of parallel planes
Identify the transversal and classify each angle pair.

10. \( \angle 5 \) and \( \angle 2 \)  
    \( p \), corresponding angles

11. \( \angle 6 \) and \( \angle 3 \)  
    \( q \), alternate interior angles

12. \( \angle 2 \) and \( \angle 4 \)  
    \( p \), alternate exterior angles

13. \( \angle 1 \) and \( \angle 2 \)  
    \( r \), same-side interior angles

---

### Example

Find each angle measure.

14. \( m\angle WYZ \)

By the Same-Side Interior Angles Theorem, 
\[(6x + 10) + (4x + 20) = 180.\]

\[x = 15 \quad \text{Solve for } x.\]

Substitute the value for \( x \) into the expression for \( m\angle TUV \).
\[m\angle TUV = 4(15) + 20 = 80^\circ\]

15. \( m\angle KLM \)

By the Corresponding Angles Postulate, 
\[8x + 28 = 10x + 4.\]

\[x = 12 \quad \text{Solve for } x.\]

Substitute the value for \( x \) into the expression for one of the obtuse angles.
\[10(12) + 4 = 124^\circ\]

\( \angle ABC \) is supplementary to the 124\(^{\circ}\) angle, so 
\[m\angle ABC = 180 - 124 = 56^\circ.\]
**Chapter 3 Parallel and Perpendicular Lines**

### Example 3-3: Proving Lines Parallel (pp. 162–169)

**Use the given information and theorems and postulates you have learned to show that \( p \parallel q \).**

- \( \angle 2 + \angle 3 = 180° \)
  \( \angle 2 \) and \( \angle 3 \) are supplementary, so \( p \parallel q \) by the Converse of the Same-Side Interior Angles Theorem.

- \( \angle 8 \cong \angle 6 \)
  \( \angle 8 \cong \angle 6 \), so \( p \parallel q \) by the Converse of the Corresponding Angles Postulate.

- \( m\angle 1 = (7x - 3)°, m\angle 5 = 5x + 15, x = 9 \)
  \( m\angle 1 = 60°, m\angle 5 = 60° \). So \( \angle 1 \cong \angle 5 \).
  \( p \parallel q \) by the Converse of the Alternate Exterior Angles Theorem.

### Example 3-4: Perpendicular Lines (pp. 172–178)

**Use the given information and theorems and postulates you have learned to show that \( c \parallel d \).**

- \( m\angle 4 = 58°, m\angle 6 = 58° \)
- \( m\angle 1 = (23x + 38)°, m\angle 5 = (17x + 56)°, x = 3 \)
- \( m\angle 6 = (12x + 6)°, m\angle 3 = (21x + 9)°, x = 5 \)
- \( m\angle 1 = 99°, m\angle 7 = (13x + 8)°, x = 7 \)

**EXERCISES**

22. Name the shortest segment from point \( K \) to \( \overline{LN} \).
23. Write and solve an inequality for \( x \).
24. Given: \( AD \parallel BC, AD \perp AB, DC \perp BC \)
   Prove: \( AB \parallel CD \)
3-5 Slopes of Lines (pp. 182–187)

**Examples**

- Use the slope formula to determine the slope of the line.
  ![Graph showing line with slope formula](image)
  \[
  \text{slope of } WX = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{2 - (-4)} = \frac{6}{6} = 1
  \]

- Use slopes to determine whether \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are parallel, perpendicular, or neither for \( A(-1, 5), B(-3, 4), C(3, -1), \) and \( D(4, -3) \).
  \[
  \text{slope of } \overrightarrow{AB} = \frac{4 - 5}{-3 - (-1)} = \frac{1}{2}
  \]
  \[
  \text{slope of } \overrightarrow{CD} = \frac{-3 - (-1)}{4 - 3} = -2
  \]
  The slopes are opposite reciprocals, so the lines are perpendicular.

**Exercises**

Use the slope formula to determine the slope of each line.

25. \[
  y = 2x - 4
  \]

26. \[
  y = 4x + 2
  \]

Use slopes to determine if the lines are parallel, perpendicular, or neither.

27. \( \overrightarrow{EF} \) and \( \overrightarrow{GH} \) for \( E(8, 2), F(-3, 4), G(6, 1), \) and \( H(-4, 3) \)

28. \( \overrightarrow{JK} \) and \( \overrightarrow{LM} \) for \( J(4, 3), K(-4, -2), L(5, 6), \) and \( M(-3, 1) \)

29. \( \overrightarrow{ST} \) and \( \overrightarrow{UV} \) for \( S(-4, 5), T(2, 3), U(3, 1), \) and \( V(4, 4) \)

3-6 Lines in the Coordinate Plane (pp. 190–197)

**Examples**

- Write the equation of the line through \( (5, -2) \) with slope \( \frac{3}{5} \) in slope-intercept form.
  \[
  y - (-2) = \frac{3}{5}(x - 5) \quad \text{Point-slope form}
  \]
  \[
  y + 2 = \frac{3}{5}x - 3 \quad \text{Simplify.}
  \]
  \[
  y = \frac{3}{5}x - 5 \quad \text{Solve for } y.
  \]

- Determine whether the lines \( y = 4x + 6 \) and \( 8x - 2y = 4 \) are parallel, intersect, or coincide.
  Solve the second equation for \( y \) to find the slope-intercept form.
  \[
  8x - 2y = 4
  \]
  \[
  y = 4x - 2
  \]
  Both the lines have a slope of 4 and have different \( y \)-intercepts, so they are parallel.

**Exercises**

Write the equation of each line in the given form.

30. the line through \( (6, 1) \) and \( (-3, 5) \) in slope-intercept form

31. the line through \( (-3, -4) \) with slope \( \frac{2}{3} \) in slope-intercept form

32. the line with \( x \)-intercept 1 and \( y \)-intercept \(-2\) in point-slope form

Determine whether the lines are parallel, intersect, or coincide.

33. \(-3x + 2y = 5, 6x - 4y = 8\)

34. \( y = 4x - 3, 5x + 2y = 1\)

35. \( y = 2x + 1, 2x - y = -1\)
Identify each of the following.
1. a pair of parallel planes
2. a pair of parallel segments
3. a pair of skew segments

Find each angle measure.
4. \( \angle 1 \)\( = (3x + 21)^\circ \), \( \angle 2 \)\( = (4x + 9)^\circ \)
5. \( \angle 3 \)\( = (20x + 17)^\circ \), \( \angle 4 \)\( = (26x - 7)^\circ \)
6. \( \angle 5 \)\( = (42x - 9)^\circ \), \( \angle 6 \)\( = (35x + 12)^\circ \)

Use the given information and the theorems and postulates you have learned to show \( f \parallel g \).
7. \( \angle 4 = (16x + 20)^\circ \), \( \angle 5 = (12x + 32)^\circ \), \( x = 3 \)
8. \( \angle 3 = (18x + 6)^\circ \), \( \angle 5 = (21x + 18)^\circ \), \( x = 4 \)

Write a two-column proof.
9. Given: \( \angle 1 \cong \angle 2 \), \( n \perp \ell \)
   Prove: \( n \perp m \)

Use the slope formula to determine the slope of each line.
10. \( \frac{W}{x} \)
11. \( \frac{y}{x} \)
12. \( \frac{y}{x} \)

13. Greg is on a 32-mile bicycle trail from Elroy, Wisconsin, to Sparta, Wisconsin. He leaves Elroy at 9:30 A.M. and arrives in Sparta at 2:00 P.M. Graph the line that represents Greg’s distance from Elroy at a given time. Find and interpret the slope of the line.

14. Graph \( \overline{QR} \) and \( \overline{ST} \) for \( Q(3, 3) \), \( R(6, -5) \), \( S(-4, 6) \), and \( T(-1, -2) \). Use slopes to determine whether the lines are parallel, perpendicular, or neither.

15. Write the equation of the line through \((-2, -5)\) with slope \(-\frac{3}{4}\) in point-slope form.

16. Determine whether the lines \(6x + y = 3\) and \(2x + 3y = 1\) are parallel, intersect, or coincide.
FOCUS ON ACT

When you take the ACT Mathematics Test, you receive a separate subscore for each of the following areas:
• Pre-Algebra/Elementary Algebra,
• Intermediate Algebra/Coordinate Geometry, and
• Plane Geometry/Trigonometry.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

1. Which of the following is an equation of the line that passes through the point $(2, -3)$ and is parallel to the line $4x - 5y = 1$?
   (A) $-4x + 5y = -23$
   (B) $-5x - 4y = 2$
   (C) $-2x - 5y = 11$
   (D) $-4x - 5y = 7$
   (E) $-5x + 4y = -22$

2. In the figure below, line $t$ crosses parallel lines $\ell$ and $m$. Which of the following statements are true?

3. In the standard $(x, y)$ coordinate plane, the line that passes through $(1, -7)$ and $(-8, 5)$ is perpendicular to the line that passes through $(3, 6)$ and $(-1, b)$. What is the value of $b$?
   (A) 2
   (B) 3
   (C) 7
   (D) 9
   (E) 10

4. Lines $m$ and $n$ are cut by a transversal so that $\angle 2$ and $\angle 5$ are corresponding angles. If $m\angle 2 = (x + 18)^\circ$ and $m\angle 5 = (2x - 28)^\circ$, which value of $x$ makes lines $m$ and $n$ parallel?
   (F) $\frac{31}{3}$
   (G) $33\frac{1}{3}$
   (H) 46
   (J) $63\frac{1}{3}$
   (K) 72

5. What is the distance between point $G(4, 2)$ and the line through the points $E(1, -2)$ and $F(7, -2)$?
   (A) 3
   (B) 4
   (C) 5
   (D) 6
   (E) 7
Short Response: Write Short Responses

Short response test items are designed to test mathematical understanding. In your response, you have to show your work and possibly describe your reasoning to show that you understand the concept. Scores are based on a 2-point scoring rubric.

Some short response questions require you to draw and label a diagram. Make sure you draw the figure as described in the problem statement and provide all markings and labeling as needed.

**EXAMPLE 1**

**Short Response** Draw and label $\angle ABC$ and $\angle CBD$, a pair of adjacent, supplementary angles. Then draw a line perpendicular to line $AD$ though point $B$ and name two right angles.

**Scoring Rubric**

2 points: The student shows an understanding of adjacent and supplementary angles and perpendicular lines. The diagram is correct, and all labels and markings are included. The student correctly names two right angles.

1 point: The student correctly sketches the diagram, but labels it incorrectly, does not name two right angles, or incorrectly names two right angles. OR the student makes minor flaws in the diagram but correctly names two right angles.

0 points: The diagram is completely incorrect, or the student gives no response.

Here are examples of how different responses were scored using the scoring rubric shown.

2-point response:

![Diag01.png](attachment:Diag01.png)

Notice that the diagram is correct and all labels and markings are included. Student correctly identified two right angles.

1-point response:

![Diag02.png](attachment:Diag02.png)

Notice that the diagram is almost correct, but points $B$ and $C$ are mislabeled. Also, the student only identified one right angle, not two.

0-point response:

![Diag03.png](attachment:Diag03.png)

Notice that the student did not complete the required diagram, and it appears to be completely incorrect.
Read each test item, and use the scoring rubric to answer each question.

Scoring Rubric

2 points: The student demonstrates an understanding of the concept, correctly answers the question, and provides a complete explanation.

1 point: The student correctly answers the question but does not show all work or does not provide an explanation.

1 point: The student makes minor errors resulting in an incorrect solution but shows an understanding of the concept.

0 points: The student gives a response showing no work or explanation, or the student gives no response.

Item A

Short Response Find \( m \angle JKM \). Identify any postulates used to determine the answer.

1. What should be included in a student’s response in order to receive 2 points?

2. A student wrote this response:

   \[
   \begin{align*}
   \text{The angles look like they have the same measure, so:} \\
   (x+65)^\circ = (3x-28)^\circ \\
   -2x = -93 \\
   x = 46.5 \\
   (3x-28) = 3(46.5) - 28 = 111.5 \\
   m \angle JKM = 111.5^\circ
   \end{align*}
   \]

What score should this response receive?
What needs to be added to the response, if anything, in order to receive 2 points?

Item B

Short Response Write a paragraph proof.

Given: \( YT \parallel ZW, XZ \perp ZW \)

Prove: \( XY \perp YT \)

So far, Issac has these thoughts written down on his paper.

3. What information is NOT necessary for Isaac to include in his proof?

4. Rewrite Issac’s paragraph so that it would receive 2 points.
CUMULATIVE ASSESSMENT, CHAPTERS 1–3

Multiple Choice

Use the diagram below for Items 1 and 2.

1. What type of angle pair are \( \angle JKM \) and \( \angle KMN \)?
   - A. Corresponding angles
   - B. Alternate exterior angles
   - C. Same-side interior angles
   - D. Alternate interior angles

2. What is \( m \angle KML \)?
   - F. 57°
   - G. 80°
   - H. 102°
   - I. 125°

3. What is a possible value of \( x \) in the diagram?
   - \( \triangle ABC \)
   - A. 2
   - B. 3
   - C. 4
   - D. 5

4. A graphic artist used a computer illustration program to draw a line connecting points with coordinates \((3, -1)\) and \((4, 6)\). She needs to draw a second line parallel to the first line. What slope should the second line have?
   - F. \( \frac{1}{7} \)
   - G. \( \frac{1}{5} \)
   - H. 5
   - I. 7

5. Which term describes a pair of vertical angles that are also supplementary?
   - A. Acute
   - B. Obtuse
   - C. Right
   - D. Straight

6. What is the equation of the line that passes through the points \((-1, 8)\) and \((4, -2)\)?
   - F. \( y = -2x + 6 \)
   - G. \( y = -\frac{1}{2}x \)
   - H. \( y = \frac{1}{2}x - 4 \)
   - I. \( y = 2x + 10 \)

7. Given the points \( R(-5, 3) \), \( S(-5, 4) \), \( T(-3, 4) \), and \( U(-3, 1) \), which line is perpendicular to \( \overline{TU} \)?
   - A. \( \overline{RS} \)
   - B. \( \overline{ST} \)
   - C. \( \overline{RT} \)
   - D. \( \overline{SU} \)

8. Which of the following is true if \( \overline{XY} \) and \( \overline{UV} \) are skew?
   - F. \( \overline{XY} \) and \( \overline{UV} \) are coplanar.
   - G. \( \overline{XY} \) and \( \overline{UV} \) are noncoplanar.
   - H. \( \overline{XY} \parallel \overline{UV} \)
   - I. \( \overline{XY} \perp \overline{UV} \)

Make sure that you answer the question that is asked. Some problems require more than one step. You must perform all of the steps to get the correct answer.

9. Point \( C \) is the midpoint of \( \overline{AB} \) for \( A(1, -2) \) and \( B(7, 2) \). What is the length of \( \overline{AC} \)? Round to the nearest tenth.
   - A. 3.0
   - B. 3.6
   - C. 5.0
   - D. 7.2

Use the diagram below for Items 10 and 11.

10. \( \overline{AD} \) bisects \( \angle CAE \), and \( \overline{AE} \) bisects \( \angle CAF \).
    If \( m \angle DAF = 120^\circ \), what is \( m \angle DAE \)?
    - F. 40°
    - G. 60°
    - H. 80°
    - I. 100°

11. What is the intersection of \( \overline{AF} \) and \( \overline{AD} \)?
    - A. \( A \)
    - B. \( F \)
    - C. \( D \)
    - D. \( \angle DAF \)
12. Which statement is true by the Transitive Property of Equality?
   - If \( x + 3 = y \), then \( y = x + 3 \).
   - If \( k = 6 \), then \( 2k = 12 \).
   - If \( a = b \) and \( b = 8 \), then \( a = 8 \).
   - If \( m = n \), then \( m + 7 = n + 7 \).

13. Which condition guarantees that \( r \parallel s \)?
   - \( \angle 1 \cong \angle 2 \)
   - \( \angle 2 \cong \angle 7 \)
   - \( \angle 1 \cong \angle 4 \)
   - \( \angle 2 \cong \angle 3 \)

14. What is the converse of the following statement?
   If \( x = 2 \), then \( x + 3 = 5 \).
   - If \( x \neq 2 \), then \( x + 3 = 5 \).
   - If \( x = 2 \), then \( x + 3 \neq 5 \).
   - If \( x + 3 \neq 5 \), then \( x \neq 2 \).
   - If \( x + 3 = 5 \), then \( x = 2 \).

Short Response
21. Given \( \ell \parallel m \) with transversal \( t \), explain why \( \angle 1 \) and \( \angle 8 \) are supplementary.

22. Read the following conditional statement.
   If two angles are vertical angles, then they are congruent.
   a. Write the converse of this conditional statement.
   b. Give a counterexample to show that the converse is false.

23. Assume that the following statements are true when the bases are loaded in a baseball game.
   If a batter hits the ball over the fence, then the batter hits a home run.
   A batter hits a home run if and only if the result is four runs scored.
   a. If a batter hits the ball over the fence when the bases are loaded, can you conclude that four runs were scored? Explain your answer.
   b. If a batter hits a home run when the bases are loaded, can you conclude that the batter hit the ball over the fence? Explain your answer.

Extended Response
24. A car passes through a tollbooth at 8:00 A.M. and begins traveling east at an average speed of 45 miles per hour. A second car passes through the same tollbooth an hour later and begins traveling east at an average speed of 60 miles per hour.
   a. Write an equation for each car that relates the number of hours \( x \) since 8:00 A.M. to the distance in miles \( y \) the car has traveled. Explain what the slope of each equation represents.
   b. Graph the system of equations on the coordinate plane.
   c. If neither car stops, at what time will the second car catch up to the first car? Explain how you determined your answer.