Transformations, lines of symmetry, and tessellations can be seen in artwork, nature, interior design, quilts, amusement parks, and marching band performances. These geometric procedures and characteristics make objects more visually pleasing.

You will learn how quilts are created by using transformations in Lesson 9-3.
Prerequisite Skills  To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 9.

For Lessons 9-1 through 9-5

Graph each pair of points.  (For review, see pages 728 and 729.)

1.  $A(1, 3), B(-1, 3)$
2.  $C(-3, 2), D(-3, -2)$
3.  $E(-2, 1), F(-1, -2)$
4.  $G(2, 5), H(5, -2)$
5.  $J(-7, 10), K(-6, 7)$
6.  $L(3, -2), M(6, -4)$

For Lesson 9-6

Find $m\angle A$. Round to the nearest tenth.  (For review, see Lesson 7-4.)

7.  $\tan A = \frac{3}{4}$
8.  $\tan A = \frac{5}{8}$
9.  $\sin A = \frac{2}{3}$
10.  $\sin A = \frac{4}{5}$
11.  $\cos A = \frac{9}{12}$
12.  $\cos A = \frac{15}{17}$

For Lesson 9-7

Find each product.  (For review, see pages 752 and 753.)

13.  $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 4 \\ -5 & -1 \end{bmatrix}$
14.  $\begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -2 \\ -2 & 3 \end{bmatrix}$
15.  $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & 4 & 5 \\ -2 & -5 & 1 \end{bmatrix}$
16.  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -3 & -3 \\ -3 & 2 \end{bmatrix}$

Transformations  Make this Foldable to help you organize your notes. Begin with one sheet of notebook paper.

Step 1  Fold

Fold a sheet of notebook paper in half lengthwise.

Step 2  Cut

Cut on every third line to create 8 tabs.

Step 3  Label

Label each tab with a vocabulary word from this chapter.

Reading and Writing  As you read and study the chapter, use each tab to write notes and examples of transformations, tessellations, and vectors on the coordinate plane.
Transformations

In a plane, you can slide, flip, turn, enlarge, or reduce figures to create new figures. These corresponding figures are frequently designed into wallpaper borders, mosaics, and artwork. Each figure that you see will correspond to another figure. These corresponding figures are formed using transformations.

A transformation maps an initial figure, called a preimage, onto a final figure, called an image. Below are some of the types of transformations. The red lines show some corresponding points.

- **translation**: A figure can be slid in any direction.
- **reflection**: A figure can be flipped over a line.
- **rotation**: A figure can be turned around a point.
- **dilation**: A figure can be enlarged or reduced.

**Exercises** Identify the following transformations. The blue figure is the preimage.

1. ![Smiley face]  
2. ![Heart]  
3. ![Parallelogram]  
4. ![Triangle]  
5. ![Rectangular box]  
6. ![Lightning bolt]  
7. ![Star]  
8. ![Symmetrical shape]  
9. ![Symmetrical shapes]  
10. ![Symmetrical shapes]

**Make a Conjecture**

11. An isometry is a transformation in which the resulting image is congruent to the preimage. Which transformations are isometries?
**DRAW REFLECTIONS** A reflection is a transformation representing a flip of a figure. Figures may be reflected in a point, a line, or a plane.

The figure shows a reflection of $ABCDE$ in line $m$. Note that the segment connecting a point and its image is perpendicular to line $m$ and is bisected by line $m$. Line $m$ is called the **line of reflection** for $ABCDE$ and its image $A'B'C'D'E'$. Because $E$ lies on the line of reflection, its preimage and image are the same point. $A'$, $A''$, $A'''$, and so on, name corresponding points for one or more transformations.

It is possible to reflect a preimage in a point. In the figure below, polygon $UVWXYZ$ is reflected in point $P$.

Note that $P$ is the midpoint of each segment connecting a point with its image.

- $\overline{UP} \equiv \overline{PU'}$, $\overline{VP} \equiv \overline{PV'}$,
- $\overline{WP} \equiv \overline{PW'}$, $\overline{XP} \equiv \overline{PX'}$,
- $\overline{YP} \equiv \overline{PY'}$, $\overline{ZP} \equiv \overline{PZ'}$

When reflecting a figure in a line or in a point, the image is congruent to the preimage. Thus, a reflection is a **congruence transformation**, or an **isometry**. That is, reflections preserve distance, angle measure, betweenness of points, and collinearity. In the figure above, polygon $UVWXYZ \equiv$ polygon $U'V'W'X'Y'Z'$. 

![Reflection Diagram](image-url)
**Example 1 Reflecting a Figure in a Line**

Draw the reflected image of quadrilateral $DEFG$ in line $m$.

**Step 1** Since $D$ is on line $m$, $D$ is its own reflection. Draw segments perpendicular to line $m$ from $E$, $F$, and $G$.

**Step 2** Locate $E'$, $F'$, and $G'$ so that line $m$ is the perpendicular bisector of $EE'$, $FF'$, and $GG'$. Points $E'$, $F'$, and $G'$ are the respective images of $E$, $F$, and $G$.

**Step 3** Connect vertices $D$, $E'$, $F'$, and $G'$.

Since points $D$, $E'$, $F'$, and $G'$ are the images of points $D$, $E$, $F$, and $G$ under reflection in line $m$, then quadrilateral $DE'F'G'$ is the reflection of quadrilateral $DEFG$ in line $m$.

Reflections can also occur in the coordinate plane.

**Example 2 Reflection in the x-axis**

**COORDINATE GEOMETRY** Quadrilateral $KLMN$ has vertices $K(2, -4)$, $L(-1, 3)$, $M(-4, 2)$, and $N(-3, -4)$. Graph $KLMN$ and its image under reflection in the x-axis. Compare the coordinates of each vertex with the coordinates of its image.

Use the vertical grid lines to find a corresponding point for each vertex so that the x-axis is equidistant from each vertex and its image.

- $K(2, -4) \rightarrow K'(2, 4)$
- $L(-1, 3) \rightarrow L'(-1, -3)$
- $M(-4, 2) \rightarrow M'(-4, -2)$
- $N(-3, -4) \rightarrow N'(-3, 4)$

Plot the reflected vertices and connect to form the image $K'L'M'N'$. The x-coordinates stay the same, but the y-coordinates are opposites. That is, $(a, b) \rightarrow (a, -b)$.

**Example 3 Reflection in the y-axis**

**COORDINATE GEOMETRY** Suppose quadrilateral $KLMN$ from Example 2 is reflected in the y-axis. Graph $KLMN$ and its image under reflection in the y-axis. Compare the coordinates of each vertex with the coordinates of its image.

Use the horizontal grid lines to find a corresponding point for each vertex so that the y-axis is equidistant from each vertex and its image.

- $K(2, -4) \rightarrow K'(-2, -4)$
- $L(-1, 3) \rightarrow L'(1, 3)$
- $M(-4, 2) \rightarrow M'(4, 2)$
- $N(-3, -4) \rightarrow N'(3, -4)$

Plot the reflected vertices and connect to form the image $K'L'M'N'$. The x-coordinates are opposites and the y-coordinates are the same. That is, $(a, b) \rightarrow (-a, b)$. 
Example 4 Reflection in the Origin

COORDINATE GEOMETRY Suppose quadrilateral $KLMN$ from Example 2 is reflected in the origin. Compare the coordinates of each vertex with the coordinates of its image.

Since $KK'$ passes through the origin, use the horizontal and vertical distances from $K$ to the origin to find the coordinates of $K'$. From $K$ to the origin is 4 units up and 2 units left. $K'$ is located by repeating that pattern from the origin. Four units up and 2 units left yields $K'(-2, 4)$.

$K(2, -4) \rightarrow K'(-2, 4)$
$L(-1, 3) \rightarrow L'(1, -3)$
$M(-4, 2) \rightarrow M'(4, -2)$
$N(-3, -4) \rightarrow N'(3, 4)$

Plot the reflected vertices and connect to form the image $K'L'M'N'$. Comparing coordinates shows that $(a, b) \rightarrow (-a, -b)$.

Example 5 Reflection in the Line $y = x$

COORDINATE GEOMETRY Suppose quadrilateral $KLMN$ from Example 2 is reflected in the line $y = x$. Compare the coordinates of each vertex with the coordinates of its image.

The slope of $y = x$ is 1. $KK'$ is perpendicular to $y = x$, so its slope is $-1$. From $K$ to the line $y = x$, move up three units and left three units. From the line $y = x$ move up three units and left three units to $K'(-4, 2)$.

$K(2, -4) \rightarrow K'(-4, 2)$
$L(-1, 3) \rightarrow L'(3, -1)$
$M(-4, 2) \rightarrow M'(2, -4)$
$N(-3, -4) \rightarrow N'(-4, -3)$

Plot the reflected vertices and connect to form the image $K'L'M'N'$. Comparing coordinates shows that $(a, b) \rightarrow (b, a)$.

Concept Summary

<table>
<thead>
<tr>
<th>Reflection</th>
<th>$x$-axis</th>
<th>$y$-axis</th>
<th>origin</th>
<th>$y = x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preimage to Image</td>
<td>$(a, b) \rightarrow (a, -b)$</td>
<td>$(a, b) \rightarrow (-a, b)$</td>
<td>$(a, b) \rightarrow (-a, -b)$</td>
<td>$(a, b) \rightarrow (b, a)$</td>
</tr>
<tr>
<td>How to find coordinates</td>
<td>Multiply the $y$-coordinate by $-1$.</td>
<td>Multiply the $x$-coordinate by $-1$.</td>
<td>Multiply both coordinates by $-1$.</td>
<td>Interchange the $x$- and $y$-coordinates.</td>
</tr>
</tbody>
</table>

Example

<p>| | | | | |</p>
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www.geometryonline.com/extra_examples/sol
**Example 6 Use Reflections**

**GOLF** Adeel and Natalie are playing miniature golf. Adeel says that he read how to use reflections to help make a hole-in-one on most miniature golf holes. Describe how he should putt the ball to make a hole-in-one.

If Adeel tries to putt the ball directly to the hole, he will strike the border as indicated by the blue line. So, he can mentally reflect the hole in the line that contains the right border. If he puts the ball at the reflected image of the hole, the ball will strike the border, and it will rebound on a path toward the hole.

---

**LINES AND POINTS OF SYMMETRY** Some figures can be folded so that the two halves match exactly. The fold is a line of reflection called a **line of symmetry**. For some figures, a point can be found that is a common point of reflection for all points on a figure. This common point of reflection is called a **point of symmetry**.

**Lines of Symmetry**

<table>
<thead>
<tr>
<th>None</th>
<th>One</th>
<th>Two</th>
<th>More than Two</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Points of Symmetry**

- ![Diagram](image5.png): No points of symmetry.
- ![Diagram](image6.png): Each point on the figure must have an image on the figure for a point of symmetry to exist.
- ![Diagram](image7.png): A point of symmetry is the midpoint of all segments between the preimage and the image.

---

**Example 7 Draw Lines of Symmetry**

Determine how many lines of symmetry a square has. Then determine whether a square has point symmetry.

- A square has four lines of symmetry.
- A square has point symmetry. $P$ is the point of symmetry such that $AP = PA'$, $BP = PB'$, $CP = PC'$, and so on.
Check for Understanding

Concept Check

1. Find a counterexample to disprove the statement Two or more lines of symmetry for a plane figure intersect in a point of symmetry.

2. OPEN ENDED Draw a figure on the coordinate plane and then reflect it in the line \( y = x \). Label the coordinates of the preimage and the image.

3. Identify four properties that are preserved in reflections.

Guided Practice

4. Copy the figure at the right. Draw its reflected image in line \( m \).

Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

5. 

6. 

7. 

COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

8. \( \overline{AB} \) with endpoints \( A(2, 4) \) and \( B(-3, -3) \) reflected in the \( x \)-axis

9. \( \triangle ABC \) with vertices \( A(-1, 4) \), \( B(4, -2) \), and \( C(0, -3) \) reflected in the \( y \)-axis

10. \( \triangle DEF \) with vertices \( D(-1, -3) \), \( E(3, -2) \), and \( F(1, 1) \) reflected in the origin

11. \( \square GHIJ \) with vertices \( G(-1, 2) \), \( H(2, 3) \), \( I(6, 1) \), and \( J(3, 0) \) reflected in the line \( y = x \)

Application NATURE Determine how many lines of symmetry each object has. Then determine whether each object has point symmetry.

12. 

13. 

14. 

Practice and Apply

Refer to the figure at the right. Name the image of each figure under a reflection in:

- line \( \ell \)
- line \( m \)
- point \( Z \)

15. \( \overline{WX} \)  
16. \( \overline{WZ} \)  
17. \( \angle XZY \)  
18. \( T \)  
19. \( \overline{UY} \)  
20. \( \triangle YVW \)  
21. \( U \)  
22. \( \angle TXZ \)  
23. \( \triangle YUZ \)

Copy each figure. Draw the image of each figure under a reflection in line \( \ell \).

24.  
25.  
26.  

Homework Help

For Exercises | See Examples
--- | ---
15–26, 38, 39 | 1
29, 34, 40, 41 | 2
30, 33, 41 | 3
27, 28 | 4
31, 32, 40 | 5
42 | 6
35–37, 44–47 | 7

Extra Practice See page 771.
COORDINATE GEOMETRY  Graph each figure and its image under the given reflection.

27. rectangle $MNPQ$ with vertices $M(2, 3), N(2, -3), P(-2, -3),$ and $Q(-2, 3)$ in the origin

28. quadrilateral $GHIJ$ with vertices $G(-2, -2), H(2, 0), I(3, 3),$ and $J(-2, 4)$ in the origin

29. square $QRST$ with vertices $Q(-1, 4), R(2, 5), S(3, 2),$ and $T(0, 1)$ in the $x$-axis

30. trapezoid with vertices $D(4, 0), E(-2, 4), F(-2, -1),$ and $G(4, -3)$ in the $y$-axis

31. $\DeltaBCD$ with vertices $B(5, 0), C(-2, 4),$ and $D(-2, 4)$ in the line $y = x$

32. $\DeltaKLM$ with vertices $K(4, 0), L(-2, 4),$ and $M(-2, 1)$ in the line $y = 2$

33. The reflected image of $\triangle FGH$ has vertices $F'(1, 4), G'(4, 2),$ and $H'(3, -2)$. Describe the reflection in the $y$-axis.

34. The reflected image of $\triangle XYZ$ has vertices $X'(1, 4), Y'(2, 2),$ and $Z'(-2, -3)$. Describe the reflection in the line $x = -1$.

Determine how many lines of symmetry each figure has. Then determine whether the figure has point symmetry.

35.  

36.  

37.  

Copy each figure and then reflect the figure in line $m$ first and then reflect that image in line $n$. Compare the preimage with the final image.

38.  

39.  

40. **COORDINATE GEOMETRY**  Square $DEFG$ with vertices $D(-1, 4), E(2, 8), F(6, 5),$ and $G(3, 1)$ is reflected first in the $x$-axis, then in the line $y = x$. Find the coordinates of $D''E''F''G''$.

41. **COORDINATE GEOMETRY**  Triangle $ABC$ has been reflected in the $x$-axis, then the $y$-axis, then the origin. The result has coordinates $A''(4, 7), B''(10, -3),$ and $C''(-6, -8)$. Find the coordinates of $A, B$, and $C$.

42. **BILLIARDS**  Tonya is playing billiards. She wants to pocket the eight ball in the lower right pocket using the white cue ball. Copy the diagram and sketch the path the eight ball must travel after being struck by the cue ball.

43. **CRITICAL THINKING**  Show that the image of a point upon reflection in the origin is the same image obtained when reflecting a point in the $x$-axis and then the $y$-axis.
DIAMONDS For Exercises 44–47, use the following information.
Diamond jewelers offer a variety of cuts. For each top view, identify any lines or points of symmetry.
44. round cut 45. pear cut 46. heart cut 47. emerald cut

48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.
Where are reflections found in nature?
Include the following in your answer:
• three examples in nature having line symmetry, and
• an explanation of how the distance from each point above the water line relates to the image in the water.
49. The image of \( A(-2, 5) \) under a reflection is \( A'(2, -5) \). Which reflection or group of reflections was used?
I. reflected in the \( x \)-axis II. reflected in the \( y \)-axis III. reflected in the origin
A. I or III B. II and III C. I and II D. I and II, or III

50. **ALGEBRA** If \( a \star c = 2a + b + 2c \), find \( a \star c \) when \( a = 25, b = 18, \) and \( c = 45 \).
A. 176 B. 158 C. 133 D. 88

---

**Maintain Your Skills**

**Mixed Review** Write a coordinate proof for each of the following. *(Lesson 8-7)*
51. The segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.
52. The segments joining the midpoints of the sides of an isosceles trapezoid form a rhombus.

Refer to trapezoid \( ACDF \). *(Lesson 8-6)*
53. Find \( BE \).
54. Let \( \overline{XY} \) be the median of \( BCDE \). Find \( XY \).
55. Let \( \overline{WZ} \) be the median of \( ABEF \). Find \( WZ \).

Solve each \( \triangle FGH \) described below. Round angle measures to the nearest degree and side measures to the nearest tenth. *(Lesson 7-6)*
56. \( m\angle G = 53, m\angle H = 71, f = 48 \)
57. \( g = 21, m\angle G = 45, m\angle F = 59 \)
58. \( h = 13.2, m\angle F = 106, f = 14.5 \)

---

**Getting Ready for the Next Lesson** **PREREQUISITE SKILL** Find the exact length of each side of quadrilateral \( EFGH \).
*(To review the Distance Formula, see Lesson 3-3.)*
59. \( EF \) 60. \( FG \)
61. \( GH \) 62. \( HE \)
**What You’ll Learn**

- Draw translated images using coordinates.
- Draw translated images by using repeated reflections.

**Vocabulary**

- translation
- composition
- glide reflection

**How are translations used in a marching band show?**

The sights and pageantry of a marching band performance can add to the excitement of a sporting event.

The movements of each band member as they progress through the show are examples of translations.

**Translations using coordinates** A translation is a transformation that moves all points of a figure the same distance in the same direction. Translations on the coordinate plane can be drawn if you know the direction and how far the figure is moving horizontally and/or vertically. For the fixed values of \( a \) and \( b \), a translation moves every point \( P(x, y) \) of a plane figure to an image \( P'(x + a, y + b) \).

One way to symbolize a transformation is to write \((x, y) \rightarrow (x + a, y + b)\).

In the figure, quadrilateral \( DEFG \) has been translated 5 units to the left and three units down. This can be written as \((x, y) \rightarrow (x - 5, y - 3)\).

\[
\begin{align*}
D(1, 2) &\rightarrow D'(1 - 5, 2 - 3) \text{ or } D'(-4, -1) \\
E(3, 1) &\rightarrow E'(3 - 5, 1 - 3) \text{ or } E'(-2, -2) \\
F(4, -1) &\rightarrow F'(4 - 5, -1 - 3) \text{ or } F'(-1, -4) \\
G(2, 0) &\rightarrow G'(2 - 5, 0 - 3) \text{ or } G'(-3, -3)
\end{align*}
\]

**Example 1 Translations in the Coordinate Plane**

Rectangle \( PQRS \) has vertices \( P(-3, 5), Q(-4, 2), R(3, 0) \), and \( S(4, 3) \). Graph \( PQRS \) and its image for the translation \((x, y) \rightarrow (x + 8, y - 5)\).

This translation moved every point of the preimage 8 units right and 5 units down.

\[
\begin{align*}
P(-3, 5) &\rightarrow P'(-3 + 8, 5 - 5) \text{ or } P'(5, 0) \\
Q(-4, 2) &\rightarrow Q'(-4 + 8, 2 - 5) \text{ or } Q'(4, -3) \\
R(3, 0) &\rightarrow R'(3 + 8, 0 - 5) \text{ or } R'(11, -5) \\
S(4, 3) &\rightarrow S'(4 + 8, 3 - 5) \text{ or } S'(12, -2)
\end{align*}
\]

Plot the translated vertices and connect to form rectangle \( P'Q'R'S' \).
**Example 2** Repeated Translations

**ANIMATION** Computers are often used to create animation. The graph shows repeated translations that result in animation of the star. Find the translation that moves star 1 to star 2 and the translation that moves star 4 to star 5.

To find the translation from star 1 to star 2, use the coordinates at the top of each star. Use the coordinates (−5, −1) and (−3, 1) in the formula.

\[(x, y) \rightarrow (x + a, y + b)\]
\[(-5, -1) \rightarrow (-3, 1)\]

\[x + a = -3\]
\[-5 + a = -3\]
\[a = 2 \quad \text{Add 5 to each side.}\]

\[y + b = 1\]
\[-1 + b = 1\]
\[b = 2 \quad \text{Add 1 to each side.}\]

The translation is \((x, y) \rightarrow (x + 2, y + 2)\).

Use the coordinates (1, 5) and (4, 5) to find the translation from star 4 to star 5.

\[(x, y) \rightarrow (x + a, y + b)\]
\[(1, 5) \rightarrow (4, 5)\]

\[x + a = 4\]
\[1 + a = 4\]
\[a = 3 \quad \text{Subtract 1 from each side.}\]

\[y + b = 5\]
\[5 + b = 5\]
\[b = 0 \quad \text{Subtract 5 from each side.}\]

The translation is \((x, y) \rightarrow (x + 3, y)\) from star 4 to star 5 and from star 5 to star 6.

**TRANSLATIONS BY REPEATED REFLECTIONS** Another way to find a translation is to perform a reflection in the first of two parallel lines and then reflect the image in the other parallel line. A transformation made up of successive transformations is called a **composition**.

**Example 3** Find a Translation Using Reflections

In the figure, lines \(m\) and \(n\) are parallel. Determine whether the red figure is a translation image of the blue preimage, quadrilateral \(ABCD\).

Reflect quadrilateral \(ABCD\) in line \(m\). The result is the green image, quadrilateral \(A'B'C'D'\). Then reflect the green image, quadrilateral \(A'B'C'D'\) in line \(n\). The red image, quadrilateral \(A''B''C''D''\), has the same orientation as quadrilateral \(ABCD\).

Quadrilateral \(A''B''C''D''\) is the translation image of quadrilateral \(ABCD\).

Since translations are compositions of two reflections, all translations are isometries. Thus, all properties preserved by reflections are preserved by translations. These properties include betweenness of points, collinearity, and angle and distance measure.
Check for Understanding

**Concept Check**

1. **OPEN ENDED** Choose integer coordinates for any two points \(A\) and \(B\). Then describe how you could count to find the translation of point \(A\) to point \(B\).

2. **Explain** which properties are preserved in a translation and why they are preserved.

3. **FIND THE ERROR** Allie and Tyrone are describing the transformation in the drawing.

   **Allie**
   
   This is a translation right 3 units and down 2 units.

   **Tyrone**

   This is a reflection in the \(y\)-axis and then the \(x\)-axis.

Who is correct? Explain your reasoning.

**Guided Practice**

In each figure, \(m \parallel n\). Determine whether the red figure is a translation image of the blue figure. Write yes or no. Explain your answer.

4. ![Guided Practice Figure 4](image)

5. ![Guided Practice Figure 5](image)

**Application**

**COORDINATE GEOMETRY** Graph each figure and its image under the given translation.

6. \(\overline{DE}\) with endpoints \(D(-3, -4)\) and \(E(4, 2)\) under the translation \((x, y) \rightarrow (x + 1, y + 3)\)

7. \(\triangle KLM\) with vertices \(K(5, -2), L(-3, -1),\) and \(M(0, 5)\) under the translation \((x, y) \rightarrow (x - 3, y - 4)\)

8. **ANIMATION** Find the translations that move the hexagon on the coordinate plane in the order given.

   ![Animation Figure](image)

**Practice and Apply**

In each figure, \(a \parallel b\). Determine whether the red figure is a translation image of the blue figure. Write yes or no. Explain your answer.

9. ![Practice and Apply Figure 9](image)

10. ![Practice and Apply Figure 10](image)

11. ![Practice and Apply Figure 11](image)
12. \[ \begin{array}{c}
\text{COORDINATE GEOMETRY} \\
\text{Graph each figure and its image under the given translation.}
\end{array} \]

15. \[ \overline{PQ} \text{ with endpoints } P(2, -4) \text{ and } Q(4, 2) \text{ under the translation left } 3 \text{ units and up } 4 \text{ units} \]

16. \[ \overline{AB} \text{ with endpoints } A(-3, 7) \text{ and } B(-6, -6) \text{ under the translation 4 units to the right and down 2 units} \]

17. \[ \triangle MJP \text{ with vertices } M(-2, -2), J(-5, 2), \text{ and } P(0, 4) \text{ under the translation } (x, y) \rightarrow (x + 1, y + 4) \]

18. \[ \triangle EFG \text{ with vertices } E(0, -4), F(-4, -4), \text{ and } G(0, 2) \text{ under the translation } (x, y) \rightarrow (x + 2, y - 1) \]

19. \[ \text{quadrilateral } PQRS \text{ with vertices } P(1, 4), Q(-1, 4), R(-2, -4), \text{ and } S(2, -4) \text{ under the translation } (x, y) \rightarrow (x - 5, y + 3) \]

20. \[ \text{pentagon } VWXYZ \text{ with vertices } V(-3, 0), W(-3, 2), X(-2, 3), Y(0, 2), \text{ and } Z(-1, 0) \text{ under the translation } (x, y) \rightarrow (x + 4, y - 3) \]

21. \[ \text{CHESS} \text{ The bishop shown in square } f8 \text{ can only move diagonally along dark squares.} \\
\text{If the bishop is in } c1 \text{ after two moves, describe the translation.} \]

22. \[ \text{RESEARCH} \text{ Use the Internet or other resource to write a possible translation for each chess piece for a single move.} \]

23. \[ \text{MOSAICS} \text{ For Exercises 23–25, use the following information.} \\
The mosaic tiling shown on the right is a thirteenth-century Roman inlaid marble tiling. Suppose this pattern is a floor design where the length of a side of the small white equilateral triangle is 12 inches. All triangles and hexagons are regular. Describe the translations in inches represented by each line.} \\
23. \text{green line} \\
24. \text{blue line} \\
25. \text{red line} \]

26. \[ \text{CRITICAL THINKING} \text{ Triangle } TWY \text{ has vertices } T(3, -7), W(7, -4), \text{ and } Y(9, -8). \text{ Triangle } BDG \text{ has vertices } B(3, 3), D(7, 6), \text{ and } G(9, 2). \text{ If } \triangle BDG \text{ is the translation image of } \triangle TWY \text{ with respect to two parallel lines, find the equations that represent two possible parallel lines.} \]
COORDINATE GEOMETRY

Graph each figure and the image under the given translation.

27. \( \triangle PQR \) with vertices \( P(-3, -2), Q(-1, 4),\) and \( R(2, -2) \) under the translation \( (x, y) \rightarrow (x + 2, y - 4) \)

28. \( \triangle RST \) with vertices \( R(-4, -1), S(-1, 3),\) and \( T(-1, 1) \) reflected in \( y = 2 \) and then reflected in \( y = -2 \)

29. Under \( (x, y) \rightarrow (x - 4, y + 5) \), \( \triangle ABC \) has translated vertices \( A'(-8, 5), B'(2, 7),\) and \( C'(3, 1) \). Find the coordinates of \( A, B, \) and \( C \).

30. Triangle \( FGH \) is translated to \( \triangle MNP \). Given \( F(3, 9), G(1, 4), M(4, 2), \) and \( P(6, -3) \), find the coordinates of \( H \) and \( N \). Then write the coordinate form of the translation.

STUDENTS

For Exercises 31–33, refer to the graphic at the right. Each bar of the graph is made up of a boy-girl-boy unit.

31. Which categories show a boy-girl-boy unit that is translated within the bar?

32. Which categories show a boy-girl-boy unit that is reflected within the bar?

33. Does each person shown represent the same percent? Explain.

Online Research

Data Update

How much allowance do teens receive?

Visit www.geometryonline.com/data_update to learn more.

34. WRITING IN MATH

Answer the question that was posed at the beginning of the lesson.

How are translations used in a marching band show?

Include the following in your answer:
• the types of movements by band members that are translations, and
• a description of a simple pattern for a band member.

Extending the Lesson

GLIDE REFLECTION

For Exercises 35–37, use the following information.

A glide reflection is a composition of a translation and a reflection in a line parallel to the direction of the translation.

35. Is a glide reflection an isometry? Explain.

36. Triangle \( DEF \) has vertices \( D(4, 3), E(2, -2), \) and \( F(0, 1) \). Sketch the image of a glide reflection composed of the translation \( (x, y) \rightarrow (x, y - 2) \) and a reflection in the \( y \)-axis.

37. Triangle \( ABC \) has vertices \( A(-3, -2), B(-1, -3), \) and \( C(2, -1) \). Sketch the image of a glide reflection composed of the translation \( (x, y) \rightarrow (x + 3, y) \) and a reflection in \( y = 1 \).
38. Triangle XYZ with vertices X(5, 4), Y(3, 1), and Z(0, 2) is translated so that X' is at (3, 1). State the coordinates of Y' and Z'.

A. Y'(5, 2) and Z'(2, 5)  
B. Y'(0, 3) and Z'(-3, 0)  
C. Y'(1, -4) and Z'(-2, -1)  
D. Y'(11, 4) and Z'(8, 6)

39. **ALGEBRA** Find the slope of a line through P(-2, 5) and T(2, -1).

A. $-\frac{3}{2}$  
B. -1  
C. 0  
D. $\frac{2}{3}$

**Maintain Your Skills**

**Mixed Review** Copy each figure. Draw the reflected image of each figure in line m. **(Lesson 9-1)**

40.  
41.  
42.  

Name the missing coordinates for each quadrilateral. **(Lesson 8-7)**

43. QRST is an isosceles trapezoid.  
44. ABCD is a parallelogram.

45. **LANDSCAPING** Juanna is a landscaper. She wishes to determine the height of a tree. Holding a drafter’s 45° triangle so that one leg is horizontal, she sights the top of the tree along the hypotenuse as shown at the right. If she is 6 yards from the tree and her eyes are 5 feet from the ground, find the height of the tree. **(Lesson 7-3)**

State the assumption you would make to start an indirect proof of each statement. **(Lesson 5-3)**

46. Every shopper that walks through the door is greeted by a salesperson.
47. If you get a job, you have filled out an application.
48. If $4y + 17 \leq 41$, $y \leq 6$.
49. If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

Find the distance between each pair of parallel lines. **(Lesson 3-6)**

50. $x = -2$  
51. $y = -6$  
52. $y = 2x + 3$  
53. $y = x + 2$

54. $x = 5$  
55. $y = -1$  
56. $y = 2x - 7$  
57. $y = x - 4$

**Getting Ready for the Next Lesson**

**PREREQUISITE SKILL** Use a protractor and draw an angle for each of the following degree measures. **(To review drawing angles, see Lesson 1-4.)**

54. 30  
55. 45  
56. 52  
57. 60  
58. 105  
59. 150
**DRAW ROTATIONS** A rotation is a transformation that turns every point of a preimage through a specified angle and direction about a fixed point. The fixed point is called the **center of rotation**.

In the figure, \( R \) is the center of rotation for the preimage \( AB\!C\!D \). The measures of angles \( \angle ARA', \angle BRB', \angle CRC', \) and \( \angle DRD' \) are equal. Any point \( P \) on the preimage \( AB\!C\!D \) has an image \( P' \) on \( A'B'C'D' \) such that the measure of \( \angle PRP' \) is a constant measure. This is called the **angle of rotation**.

A rotation exhibits all of the properties of isometries, including preservation of distance and angle measure. Therefore, it is an isometry.

**Example 1** **Draw a Rotation**

Triangle \( ABC \) has vertices \( A(2, 3), B(6, 3), \) and \( C(5, 5) \). Draw the image of \( \triangle ABC \) under a rotation of 60° counterclockwise about the origin.

- First graph \( \triangle ABC \).
- Draw a segment from the origin \( O \), to point \( A \).
- Use a protractor to measure a 60° angle counterclockwise with \( OA \) as one side.
- Draw \( \overline{OR} \).
- Use a compass to copy \( \overline{OA} \) onto \( \overline{OR} \). Name the segment \( OA' \).
• Repeat with points B and C. The image of \( \triangle ABC \) under a 60° counterclockwise rotation about the origin.

Another way to perform rotations is by reflecting a figure successively in two intersecting lines. Reflecting a figure once and then reflecting the image in a second line is another example of a composition of reflections.

**Geometry Software Investigation**

**Reflections in Intersecting Lines**

**Construct a Figure**
- Use The Geometer's Sketchpad to construct scalene triangle \( ABC \).
- Construct lines \( m \) and \( n \) so that they intersect outside \( \triangle ABC \).
- Label the point of intersection \( P \).

**Analyze**
1. Reflect \( \triangle ABC \) in line \( m \). Then, reflect \( \triangle A'B'C' \) in line \( n \).
2. Describe the transformation of \( \triangle ABC \) to \( \triangle A''B''C'' \).
3. Measure the acute angle formed by \( m \) and \( n \).
4. Construct a segment from \( A \) to \( P \) and from \( A'' \) to \( P \). Find the measure of the angle of rotation, \( \angle APA'' \).
5. Find \( m \angle BPB'' \) and \( m \angle CPC'' \).

**Make a Conjecture**
6. What is the relationship between the measures of the angles of rotation and the measure of the acute angle formed by \( m \) and \( n \)?

When rotating a figure by reflecting it in two intersecting lines, there is a relationship between the angle of rotation and the angle formed by the intersecting lines.

**Key Concept**

**Postulate 9.1** In a given rotation, if \( A \) is the preimage, \( A'' \) is the image, and \( P \) is the center of rotation, then the measure of the angle of rotation \( \angle APA'' \) is twice the measure of the acute or right angle formed by the intersecting lines of reflection.

**Corollary 9.1** Reflecting an image successively in two perpendicular lines results in a 180° rotation.
**Example 2** Reflections in Intersecting Lines

Find the image of rectangle $DEFG$ under reflections in line $m$ and then line $n$.

First reflect rectangle $DEFG$ in line $m$. Then label the image $D'E'F'G'$.

Next, reflect the image in line $n$. Rectangle $D''E''F''G''$ is the image of rectangle $DEFG$ under reflections in lines $m$ and $n$. How can you transform $DEFG$ directly to $D''E''F''G''$ by using a rotation?

**ROTATIONAL SYMMETRY** Some objects have rotational symmetry. If a figure can be rotated less than 360 degrees about a point so that the image and the preimage are indistinguishable, then the figure has **rotational symmetry**.

In the figure, the pentagon has rotational symmetry of order 5 because there are 5 rotations of less than 360° (including 0 degrees) that produce an image indistinguishable from the original. The rotational symmetry has a magnitude of $72°$ because 360 degrees divided by the order, in this case 5, produces the magnitude of the symmetry.

**Example 3** Identifying Rotational Symmetry

**QUILTS** One example of rotational symmetry artwork is quilt patterns. A quilt made by Judy Mathieson of Sebastopol, California, won the Masters Award for Contemporary Craftsmanship at the International Quilt Festival in 1999.

Identify the order and magnitude of the symmetry in each part of the quilt.

a. large star in center of quilt
   The large star in the center of the quilt has rotational symmetry of order 20 and magnitude of $18°$.

b. entire quilt
   The entire quilt has rotational symmetry of order 4 and magnitude of $90°$.

**Check for Understanding**

**Concept Check**
1. OPEN ENDED Draw a figure on the coordinate plane in Quadrant I. Rotate the figure clockwise 90 degrees about the origin. Then rotate the figure 90 degrees counterclockwise. Describe the results using the coordinates.

2. Explain two techniques that can be used to rotate a figure.

3. Compare and contrast translations and rotations.

**Guided Practice**
4. Copy $\triangle BCD$ and rotate the triangle $60°$ counterclockwise about point $G$. 

![Diagram of rotated triangle]
Copy each figure. Use a composition of reflections to find the rotation image with respect to lines $\ell$ and $m$.

5. \[ \begin{align*}
  \ell & \quad \text{A} \quad \text{B} \\
  \quad \text{D} \quad \text{C} \\
\end{align*} \]

6. \[ \begin{align*}
  \ell & \quad \text{E} \quad \text{F} \\
  \quad \text{D} \quad \text{G} \\
\end{align*} \]

7. $XY$ has endpoints $X(-5, 8)$ and $Y(0, 3)$. Draw the image of $XY$ under a rotation of $45^\circ$ clockwise about the origin.

8. $\triangle PQR$ has vertices $P(-1, 8), Q(4, -2),$ and $R(-7, -4)$. Draw the image of $\triangle PQR$ under a rotation of $90^\circ$ counterclockwise about the origin.

9. Identify the order and magnitude of the rotational symmetry in a regular hexagon.

10. Identify the order and magnitude of the rotational symmetry in a regular octagon.

**Application**

11. FANS The blades of a fan exhibit rotational symmetry. Identify the order and magnitude of the symmetry of the blades of each fan in the pictures.

![Fan Images]

**Practice and Apply**

12. Copy pentagon $BCDEF$. Then rotate the pentagon $110^\circ$ counterclockwise about point $R$.

13. Copy $\triangle MNP$. Then rotate the triangle $180^\circ$ counterclockwise around point $Q$.

![Pentagon and Triangle Images]

**COORDINATE GEOMETRY** Draw the rotation image of each figure $90^\circ$ in the given direction about the center point and label the vertices.

14. $\triangle XYZ$ with vertices $X(0, -1), Y(3, 1),$ and $Z(1, 5)$ counterclockwise about the point $P(-1, 1)$

15. $\triangle RST$ with vertices $R(0, 1), S(5, 1),$ and $T(2, 5)$ clockwise about the point $P(-2, 5)$

**RECREATION** For Exercises 16–18, use the following information.

A Ferris wheel’s motion is an example of a rotation. The Ferris wheel shown has 20 cars.

16. Identify the order and magnitude of the symmetry of a 20-seat Ferris wheel.

17. What is the measure of the angle of rotation if seat 1 of a 20-seat Ferris wheel is moved to the seat 5 position?

18. If seat 1 of a 20-seat Ferris wheel is rotated $144^\circ$, find the original seat number of the position it now occupies.
Copy each figure. Use a composition of reflections to find the rotation image with respect to lines \( m \) and \( t \).

19. \[
\begin{array}{c}
X \\
Y \\
Z \\
\end{array}
\]
20. \[
\begin{array}{c}
R \\
S \\
\end{array}
\]
21. \[
\begin{array}{c}
J \\
K \\
M \\
\end{array}
\]

**COORDINATE GEOMETRY**  Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.

22. \( \triangle TUV \) with vertices \( T(4, 0), U(2, 3), \) and \( V(1, 2) \), reflected in the \( y \)-axis and then the \( x \)-axis
23. \( \triangle KLM \) with vertices \( K(5, 0), L(2, 4), \) and \( M(-2, 4) \), reflected in the line \( y = x \) and then the \( x \)-axis
24. \( \triangle XYZ \) with vertices \( X(5, 0), Y(3, 4), \) and \( Z(-3, 4) \), reflected in the line \( y = -x \) and then the line \( y = x \)

25. **COORDINATE GEOMETRY**  The point at \( (2, 0) \) is rotated \( 30^\circ \) counterclockwise about the origin. Find the exact coordinates of its image.

26. **MUSIC**  A five-disc CD changer rotates as each CD is played. Identify the magnitude of the rotational symmetry as the changer moves from one CD to another.

Determine whether the indicated composition of reflections is a rotation. Explain.

27. \[
\begin{array}{c}
A \\
B \quad C \\
A' \quad B' \quad C' \\
A'' \\
\end{array}
\]
28. \[
\begin{array}{c}
D \\
E \quad F \\
D' \quad E' \quad F' \\
D'' \\
\end{array}
\]

**AMUSEMENT RIDES**  For each ride, determine whether the rider undergoes a rotation. Write yes or no.

29. spinning teacups 30. scrambler 31. roller coaster loop

32. **ALPHABET**  Which capital letters of the alphabet produce the same letter after being rotated \( 180^\circ \)?

33. **CRITICAL THINKING**  In \( \triangle ABC \), \( m \angle BAC = 40 \). Triangle \( AB'C' \) is the image of \( \triangle ABC \) under reflection and \( \triangle AB'C' \) is the image of \( \triangle AB'C \) under reflection. How many such reflections would be necessary to map \( \triangle ABC \) onto itself?

34. **CRITICAL THINKING**  If a rotation is performed on a coordinate plane, what angles of rotation would make the rotations easier? Explain.
35. **COORDINATE GEOMETRY** Quadrilateral QRST is rotated 90° clockwise about the origin. Describe the transformation using coordinate notation.

36. Triangle FGH is rotated 80° clockwise and then rotated 150° counterclockwise about the origin. To what counterclockwise rotation about the origin is this equivalent?

**CRITICAL THINKING** For Exercises 37–39, use the following information.
Points that do not change position under a transformation are called **invariant points**. For each of the following transformations, identify any invariant points.

37. reflection in a line
38. a rotation of \(\theta\) (0 < \(\theta\) < 360) about point \(P\)
39. \((x, y) \rightarrow (x + a, y + b)\), where \(a\) and \(b\) are not 0

40. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How do amusement rides exemplify rotations?**
Include the following in your answer:
- a description of how the Tilt-A-Whirl actually rotates two ways, and
- other amusement rides that use rotation.

41. In the figure, describe the rotation that moves triangle 1 to triangle 2.

- A 180° clockwise
- B 135° clockwise
- C 135° counterclockwise
- D 90° counterclockwise

42. **ALGEBRA** Suppose \(x\) is \(\frac{2}{5}\) of \(y\) and \(y\) is \(\frac{1}{3}\) of \(z\). If \(x = 6\), then \(z = ?\)

- A \(\frac{4}{5}\)
- B \(\frac{18}{5}\)
- C 5
- D 45

**Extending the Lesson**

A **direct isometry** is one in which the image of a figure is found by moving the figure intact within the plane. An **indirect isometry** cannot be performed by maintaining the clockwise orientation of the points as in a direct isometry.

43. Copy and complete the table below. Determine whether each transformation preserves the given property. Write **yes** or **no**.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>angle measure</th>
<th>betweenness of points</th>
<th>orientation</th>
<th>collinearity</th>
<th>distance measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>reflection</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>translation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rotation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Identify each type of transformation as a direct isometry or an indirect isometry.

44. reflection
45. translation
46. rotation
**Practice Quiz 1**

**Graph each figure and the image in the given reflection.** *(Lesson 9-1)*

1. \(\triangle DEF\) with vertices \(D(-1, 1), E(1, 4)\) and \(F(3, 2)\) in the origin

2. quadrilateral \(ABCD\) with vertices \(A(0, 2), B(2, 2), C(3, 0)\), and \(D(-1, 1)\) in the line \(y = x\)

**Graph each figure and the image under the given translation.** *(Lesson 9-2)*

3. \(\overline{PQ}\) with endpoints \(P(1, -4)\) and \(Q(4, -1)\) under the translation left 3 units and up 4 units

4. \(\triangle KLM\) with vertices \(K(-2, 0), L(-4, 2)\), and \(M(0, 4)\) under the translation \((x, y) \rightarrow (x + 1, y - 4)\)

5. Identify the order and magnitude of the symmetry of a 36-horse carousel. *(Lesson 9-3)*

---

**Maintain Your Skills**

**Mixed Review** In each figure, \(a \parallel b\). Determine whether the blue figure is a translation image of the red figure. Write yes or no. Explain your answer. *(Lesson 9-2)*

- 47.
- 48.
- 49.

For the figure to the right, name the reflected image of each image. *(Lesson 9-1)*

- 50. \(\overline{AG}\) in line \(p\)
- 51. \(F\) in point \(G\)
- 52. \(\overline{GE}\) in line \(q\)
- 53. \(\angle CGD\) in line \(p\)

Complete each statement about \(\square PQRS\). Justify your answer. *(Lesson 8-2)*

- 54. \(\overline{QR} \parallel \) __________
- 55. \(\overline{PT} \equiv \) __________
- 56. \(\angle SQR \equiv \) __________
- 57. \(\angle QPS \equiv \) __________

58. **SURVEYING** A surveyor is 100 meters from a building and finds that the angle of elevation to the top of the building is 23°. If the surveyor’s eye level is 1.55 meters above the ground, find the height of the building. *(Lesson 7-5)*

Determine whether the given measures can be the lengths of the sides of a triangle. Write yes or no. *(Lesson 5-4)*

- 59. 6, 8, 16
- 60. 12, 17, 20
- 61. 22, 23, 37

**Getting Ready for the Next Lesson** **PREREQUISITE SKILL** Find whole number values for each variable so each equation is true. *(To review solving problems by making a table, see pages 737–738.)*

- 62. \(180a = 360\)
- 63. \(180a + 90b = 360\)
- 64. \(135a + 45b = 360\)
- 65. \(120a + 30b = 360\)
- 66. \(180a + 60b = 360\)
- 67. \(180a + 30b = 360\)

---

**Practice Quiz 1**

**Lesson 9-1 through 9-3**
Tessellations

Virginia SOL Standard: G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

What You’ll Learn

- Identify regular tessellations.
- Create tessellations with specific attributes.

How are tessellations used in art?

M.C. Escher (1898–1972) was a Dutch graphic artist famous for repeating geometric patterns. He was also well known for his spatial illusions, impossible buildings, and techniques in woodcutting and lithography. In the picture, figures can be reduced to basic regular polygons. Equilateral triangles and regular hexagons are prominent in the repeating patterns.

REGULAR TESSELLATIONS Reflections, translations, and rotations can be used to create patterns using polygons. A pattern that covers a plane by transforming the same figure or set of figures so that there are no overlapping or empty spaces is called a tessellation.

In a tessellation, the sum of the measures of the angles of the polygons surrounding a point (at a vertex) is 360.

You can use what you know about angle measures in regular polygons to help determine which polygons tessellate.

Reading Math

The word tessellation comes from the Latin word tessera which means "a square tablet." These small square pieces of stone or tile were used in mosaics.

Symmetry Drawing E103, M.C. Escher

Study Tip

The word tessellation comes from the Latin word tessera which means "a square tablet." These small square pieces of stone or tile were used in mosaics.

Geometry Activity

Tessellations of Regular Polygons

Model and Analyze

- Study a set of pattern blocks to determine which shapes are regular.
- Make a tessellation with each type of regular polygon.

1. Which shapes in the set are regular?
2. Write an expression showing the sum of the angles at each vertex of the tessellation.
3. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Regular Polygon</th>
<th>triangle</th>
<th>square</th>
<th>pentagon</th>
<th>hexagon</th>
<th>heptagon</th>
<th>octagon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure of One Interior Angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does it tessellate?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Make a Conjecture

4. What must be true of the angle measure of a regular polygon for a regular tessellation to occur?
The tessellations you formed in the Geometry Activity are regular tessellations. A regular tessellation is a tessellation formed by only one type of regular polygon. In the activity, you found that if a regular polygon has an interior angle with a measure that is a factor of 360, then the polygon will tessellate the plane.

**Example 1**

Determine whether a regular 24-gon tessellates the plane. Explain.

Let $\angle 1$ represent one interior angle of a regular 24-gon.

$$m\angle 1 = \frac{180(n - 2)}{n} \quad \text{Interior Angle Formula}$$

$$= \frac{180(24 - 2)}{24} \quad \text{Substitution}$$

$$= 165 \quad \text{Simplify.}$$

Since 165 is not a factor of 360, a 24-gon will not tessellate the plane.

**TESSELLATIONS WITH SPECIFIC ATTRIBUTES**

A tessellation pattern can contain any type of polygon. Tessellations containing the same arrangement of shapes and angles at each vertex are called uniform.

<table>
<thead>
<tr>
<th>uniform</th>
<th>not uniform</th>
</tr>
</thead>
<tbody>
<tr>
<td>At vertex $A$, there are four congruent angles.</td>
<td>At vertex $A$, there are three angles that are all congruent.</td>
</tr>
<tr>
<td>At vertex $B$, there are the same four congruent angles.</td>
<td>At vertex $B$, there are five angles; four are congruent and one is different.</td>
</tr>
<tr>
<td>At vertex $A$, there are four angles that consist of two congruent pairs.</td>
<td>At vertex $A$, there are eight congruent angles.</td>
</tr>
<tr>
<td>At vertex $B$, there are the same two congruent pairs.</td>
<td>At vertex $B$, there are four congruent angles.</td>
</tr>
</tbody>
</table>

Tessellations can be formed using more than one type of polygon. A uniform tessellation formed using two or more regular polygons is called a semi-regular tessellation.
**Example 2** Semi-Regular Tessellation

Determine whether a semi-regular tessellation can be created from regular hexagons and equilateral triangles, all having sides 1 unit long.

**Method 1** Make a model.

Two semi-regular models are shown. You will notice that the spaces at each vertex can be filled in with equilateral triangles. Model 1 has two hexagons and two equilateral triangles arranged in an alternating pattern around each vertex. Model 2 has one hexagon and four equilateral triangles at each vertex.

**Method 2** Solve algebraically.

Each interior angle of a regular hexagon measures $120^\circ$.

Each angle of an equilateral triangle measures $60^\circ$. Find whole-number values for $h$ and $t$ so that $120h + 60t = 360$.

Let $h = 1$. Let $h = 2$.

\[
\begin{align*}
120(1) + 60t &= 360 & \text{Substitution} \\
120 + 60t &= 360 & \text{Simplify} \\
60t &= 240 & \text{Subtract from each side} \\
t &= 4 & \text{Divide each side by 60} \\
\end{align*}
\]

When $h = 1$ and $t = 4$, there is one hexagon with four equilateral triangles at each vertex. (Model 2)

When $h = 2$ and $t = 2$, there are two hexagons and two equilateral triangles. (Model 1)

Note if $h = 0$ and $t = 6$ or $h = 3$ and $t = 0$, then the tessellations are regular because there would be only one regular polygon.

---

**Example 3** Classify Tessellations

**FLOORING** Tile flooring comes in many shapes and patterns. Determine whether the pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.

The pattern is a tessellation because at the different vertices the sum of the angles is $360^\circ$.

The tessellation is uniform because at each vertex there are two squares, a triangle, and a hexagon arranged in the same order. The tessellation is also semi-regular since more than one regular polygon is used.
Determine whether each regular polygon tessellates the plane. Explain.

4. decagon
5. 30-gon

Determine whether a semi-regular tessellation can be created from each set of figures. Assume that each figure has a side length of 1 unit.

6. a square and a triangle
7. an octagon and a square

Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.

8. 
9. 

10. QUILTING The “Postage Stamp” pattern can be used in quilting. Explain why this is a tessellation and what kind it is.

**Practice and Apply**

Determine whether each regular polygon tessellates the plane. Explain.

11. nonagon
12. hexagon
13. equilateral triangle
14. dodecagon
15. 23-gon
16. 36-gon

Determine whether a semi-regular tessellation can be created from each set of figures. Assume that each figure has a side length of 1 unit.

17. regular octagons and non-square rhombi
18. regular dodecagons and equilateral triangles
19. regular dodecagons, squares, and equilateral triangles
20. regular heptagons, squares, and equilateral triangles

Determine whether each figure tessellates the plane. If so, describe the tessellation as uniform, not uniform, regular, or semi-regular.

21. parallelogram
22. kite

23. quadrilateral
24. pentagon and square

Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.

25. 
26. 
27. **Tessellations**

![Tessellation Diagram]

Determine whether each statement is *always*, *sometimes*, or *never* true. Justify your answers.

30. Any triangle will tessellate the plane.
31. Semi-regular tessellations are not uniform.
32. Uniform tessellations are semi-regular.
33. Every quadrilateral will tessellate the plane.
34. Regular 16-gons will tessellate the plane.

**INTERIOR DESIGN**

For Exercises 35 and 36, use the following information.

Kele’s family is tiling the entry floor with the tiles in the pattern shown.

35. Determine whether the pattern is a tessellation.
36. Is the tessellation *uniform*, *regular*, or *semi-regular*?

37. **BEES**

A honeycomb is composed of hexagonal cells made of wax in which bees store honey. Determine whether this pattern is a tessellation. If so, describe it as *uniform*, *not uniform*, *regular*, or *semi-regular*.

38. **CRITICAL THINKING**

What could be the measures of the interior angles in a pentagon that tessellates a plane? Is this tessellation regular? Is it uniform?

39. **WRITING IN MATH**

Answer the question that was posed at the beginning of the lesson.

How are tessellations used in art?
Include the following in your answer:
- how equilateral triangles and regular hexagons form a tessellation, and
- other geometric figures that can be found in the picture.

40. Find the measure of an interior angle of a regular nonagon.

<p>| | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>150</td>
<td>B</td>
<td>147</td>
</tr>
<tr>
<td>C</td>
<td>140</td>
<td>D</td>
<td>115</td>
</tr>
</tbody>
</table>

41. **ALGEBRA**

Evaluate \(\frac{360(x - 2)}{2x} - \frac{180}{x}\) if \(x = 12\).

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>135</td>
<td>B</td>
<td>150</td>
</tr>
<tr>
<td>C</td>
<td>160</td>
<td>D</td>
<td>225</td>
</tr>
</tbody>
</table>

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**BRICKWORK**

In the picture, suppose the side of the octagon is the same length as the side of the square. What kind of tessellation is this?
Maintain Your Skills

Mixed Review

COORDINATE GEOMETRY Draw the rotation image of each figure 90° in the given direction about the center point and label the coordinates. (Lesson 9-3)

42. ΔABC with A(8, 1), B(2, –6), and C(–4, –2) counterclockwise about P(–2, 2)
43. ΔDEF with D(6, 2), E(6, –3), and F(2, 3) clockwise about P(3, –2)
44. □GHIJ with G(–1, 2), H(–3, –3), I(–5, –6), and J(–3, –1) counterclockwise about P(–2, –3)
45. rectangle KLMN with K(–3, –5), L(3, 3), M(7, 0), and N(1, –8) counterclockwise about P(–2, 0)

46. REMODELING The diagram at the right shows the floor plan of Justin’s kitchen. Each square on the diagram represents a 3 foot by 3 foot area. While remodeling his kitchen, Justin moved his refrigerator from square A to square B. Describe the move. (Lesson 9-2)

ALGEBRA Find x and y so that each quadrilateral is a parallelogram. (Lesson 8-3)

47. 48. 49.

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple. (Lesson 7-2)

50. 12, 16, 20
51. 9, 10, 15
52. 2.5, 6, 6.5
53. 14, 14√3, 28
54. 14, 48, 50
55. \(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\)

Points A, B, and C are the midpoints of \(\overline{DF}, \overline{DE}, \) and \(\overline{EF}\), respectively. (Lesson 6-4)

56. If \(BC = 11, AC = 13,\) and \(AB = 15,\) find the perimeter of \(\triangle DEF.\)
57. If \(DE = 18, DA = 10,\) and \(FC = 7,\) find \(AB, BC,\) and \(AC.\)

COORDINATE GEOMETRY For Exercises 58–61, use the following information. The vertices of quadrilateral \(PQRS\) are \(P(5, 2), Q(1, 6), R(–3, 2),\) and \(S(1, –2).\) (Lesson 3-3)

58. Show that the opposite sides of quadrilateral \(PQRS\) are parallel.
59. Show that the adjacent sides of quadrilateral \(PQRS\) are perpendicular
60. Determine the length of each side of quadrilateral \(PQRS.\)
61. What type of figure is quadrilateral \(PQRS?\)

Getting Ready for the Next Lesson

PREREQUISITE SKILL If quadrilateral \(ABCD \sim\) quadrilateral \(WXYZ,\) find each of the following. (To review similar polygons, see Lesson 6-2.)

62. scale factor of \(ABCD\) to \(WXYZ\)
63. \(XY\)
64. \(YZ\)
65. \(WZ\)
Model and Analyze

1. Is the area of the square in Step 1 of Activity 1 the same as the area of the new shape in Step 2? Explain.

2. Describe how you would create the unit for the pattern shown at the right.

Make a tessellation for each pattern described. Use a tessellation of two rows of three squares as your base.

3.  

4.  

5.  

What You’ll Learn

- Determine whether a dilation is an enlargement, a reduction, or a congruence transformation.
- Determine the scale factor for a given dilation.

Vocabulary

- dilation
- similarity transformation

How do you use dilations when you use a computer?

Have you ever tried to paste an image into a word processing document and the picture was too large? Many word processing programs allow you to scale the size of the picture so that you can fit it in your document. Scaling a picture is an example of a dilation.

CLASSIFY DILATIONS All of the transformations you have studied so far in this chapter produce images that are congruent to the original figure. A dilation is another type of transformation.

A dilation is a transformation that may change the size of a figure. A dilation requires a center point and a scale factor. The figures below show how dilations can result in a larger figure and a smaller figure than the original.

Triangle $A'B'D'$ is a dilation of $\triangle ABD$.

$$CA' = 2(CA)$$
$$CB' = 2(CB)$$
$$CD' = 2(CD)$$

$\triangle A'B'D'$ is larger than $\triangle ABD$.

Rectangle $M'N'O'P'$ is a dilation of rectangle $MNOP$.

$$XM' = \frac{1}{3}(XM)$$
$$XN' = \frac{1}{3}(XN)$$

$\triangle A'B'D'$ is larger than $\triangle ABD$.

The value of $r$ determines whether the dilation is an enlargement or a reduction.

Key Concept

**Dilation**

- If $|r| > 1$, the dilation is an enlargement.
- If $0 < |r| < 1$, the dilation is a reduction.
- If $|r| = 1$, the dilation is a congruence transformation.
As you can see in the figures on the previous page, dilation preserves angle measure, betweenness of points, and collinearity, but does not preserve distance. That is, dilations produce similar figures. Therefore, a dilation is a **similarity transformation**.

This means that \( \triangle ABD \sim \triangle A'B'D' \) and \( \square MNOP \sim \square M'N'O'P' \). This implies that 
\[
\frac{A'B'}{AB} = \frac{B'D'}{BD} = \frac{A'D'}{AD} \quad \text{and} \quad \frac{M'N'}{MN} = \frac{N'O'}{NO} = \frac{O'P'}{OP} = \frac{M'P'}{MP}.
\]
The ratios of measures of the corresponding parts is equal to the absolute value scale factor of the dilation, \( |r| \).
So, \( |r| \) determines the size of the image as compared to the size of the preimage.

### Theorem 9.1

If a dilation with center \( C \) and a scale factor of \( r \) transforms \( A \) to \( E \) and \( B \) to \( D \), then \( ED = |r| \cdot (AB) \).

![Image](image1.png)

You will prove Theorem 9.1 in Exercise 41.

### Example 1 Determine Measures Under Dilations

Find the measure of the dilation image \( A'B' \) or the preimage \( AB \) using the given scale factor.

**a.** \( AB = 12, r = -2 \)

\[
A'B' = |r| \cdot (AB) = |2| \cdot 12 = 24
\]

**b.** \( A'B' = 36, r = \frac{1}{4} \)

\[
A'B' = |r| \cdot (AB) = \frac{1}{4} \cdot 36 = 9
\]

When the scale factor is negative, the image falls on the opposite side of the center than the preimage.

### Key Concept

**Dilations**

If \( r > 0 \), \( P' \) lies on \( \overrightarrow{CP} \), and \( CP' = r \cdot CP \).

If \( r < 0 \), \( P' \) lies on \( \overrightarrow{CP} \) the ray opposite \( \overrightarrow{CP} \), and \( CP' = |r| \cdot CP \).

The center of a dilation is always its own image.

### Example 2 Draw a Dilation

Draw the dilation image of \( \triangle JKL \) with center \( C \) and \( r = -\frac{1}{2} \).

Since \( 0 < |r| < 1 \), the dilation is a reduction of \( \triangle JKL \).

Draw \( \overline{CJ} \), \( \overline{CK} \), and \( \overline{CL} \). Since \( r \) is negative, \( J' \), \( K' \), and \( L' \) will lie on \( \overrightarrow{CJ} \), \( \overrightarrow{CK} \), and \( \overrightarrow{CL} \), respectively. Locate \( J' \), \( K' \), and \( L' \) so that \( C \) is halfway between \( J \) and \( J' \), \( C \) is halfway between \( K \) and \( K' \), and \( C \) is halfway between \( L \) and \( L' \).

Draw \( \triangle J'K'L' \).
In the coordinate plane, you can use the scale factor to determine the coordinates of the image of dilations centered at the origin.

**Theorem 9.2**

If \( P(x, y) \) is the preimage of a dilation centered at the origin with a scale factor \( r \), then the image is \( P'(rx, ry) \).

**Example 3**  
**Dilations in the Coordinate Plane**  

**COORDINATE GEOMETRY** Triangle \( ABC \) has vertices \( A(7, 10) \), \( B(4, -6) \), and \( C(-2, 3) \). Find the image of \( \triangle ABC \) after a dilation centered at the origin with a scale factor of 2. Sketch the preimage and the image.

<table>
<thead>
<tr>
<th>Preimage  ((x, y))</th>
<th>Image ((2x, 2y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A(7, 10) )</td>
<td>( A'(14, 20) )</td>
</tr>
<tr>
<td>( B(4, -6) )</td>
<td>( B'(8, -12) )</td>
</tr>
<tr>
<td>( C(-2, 3) )</td>
<td>( C'(-4, 6) )</td>
</tr>
</tbody>
</table>

**IDENTIFY THE SCALE FACTOR** In Chapter 6, you found scale factors of similar figures. If you know the measurement of a figure and its dilated image, you can determine the scale factor.

**Example 4**  
**Identify Scale Factor**

Determine the scale factor for each dilation with center \( C \). Then determine whether the dilation is an enlargement, reduction, or congruence transformation.

a.  

\[
\text{scale factor} = \frac{\text{image length}}{\text{preimage length}} = \frac{6 \text{ units}}{3 \text{ units}} = 2
\]

Since the scale factor is greater than 1, the dilation is an enlargement.

b.  

\[
\text{scale factor} = \frac{\text{image length}}{\text{preimage length}} = \frac{4 \text{ units}}{4 \text{ units}} = 1
\]

Since the scale factor is 1, the dilation is a congruence transformation.
Lesson 9-5
Dilations

Scale Drawing

Example 5

Multiple-Choice Test Item

Jacob wants to make a scale drawing of a painting in an art museum. The painting is 4 feet wide and 8 feet long. Jacob decides on a dilation reduction factor of \( \frac{1}{6} \). What size paper will he need to make a complete sketch?

- A 8 \( \frac{1}{2} \) in. by 11 in.
- B 9 in. by 12 in.
- C 11 in. by 14 in.
- D 11 in. by 17 in.

Read the Test Item

The painting’s dimensions are given in feet, and the paper choices are in inches. You need to convert from feet to inches in the problem.

Solve the Test Item

Step 1 Convert feet to inches.
- 4 feet = 4(12) or 48 inches
- 8 feet = 8(12) or 96 inches

Step 2 Use the scale factor to find the image dimensions.
- \( w = \frac{1}{6}(48) \) or 8
- \( \ell = \frac{1}{6}(96) \) or 16

Step 3 The dimensions of the image are 8 inches by 16 inches. Choice D is the only size paper on which the scale drawing will fit.

Check for Understanding

Concept Check
1. Find a counterexample to disprove the statement *All dilations are isometries.*

2. OPEN ENDED Draw a figure on the coordinate plane. Then show a dilation of the figure that is a reduction and a dilation of the figure that is an enlargement.

3. FIND THE ERROR Desiree and Trey are trying to describe the effect of a negative \( r \) value for a dilation of quadrilateral \( WXYZ \).

Who is correct? Explain your reasoning.

Guided Practice
Draw the dilation image of each figure with center \( C \) and the given scale factor.

4. \( r = 4 \)
5. \( r = \frac{1}{5} \)
6. \( r = -2 \)
Find the measure of the dilation image $AB'$ or the preimage $AB$ using the given scale factor.

7. $AB = 3, r = 4$
8. $A'B' = 8, r = -\frac{2}{5}$

9. $PQ$ has endpoints $P(9, 0)$ and $Q(0, 6)$. Find the image of $PQ$ after a dilation centered at the origin with a scale factor $r = \frac{1}{3}$. Sketch the preimage and the image.

10. Triangle $KLM$ has vertices $K(5, 8), L(-3, 4)$, and $M(-1, -6)$. Find the image of $\triangle KLM$ after a dilation centered at the origin with scale factor of 3. Sketch the preimage and the image.

Determine the scale factor for each dilation with center $C$. Determine whether the dilation is an enlargement, reduction, or congruence transformation.

11. $ST = 6, r = 3$
12. $S'T' = 12, r = \frac{2}{3}$
13. Alexis made a scale drawing of the plan for her spring garden. It will be a rectangle measuring 18 feet by 12 feet. On the scaled version, it measures 8 inches on the longer sides. What is the measure of each of the shorter sides?

14. $r = 3$
15. $r = 2$
16. $r = \frac{1}{2}$
17. $r = \frac{2}{5}$
18. $r = -1$
19. $r = -\frac{1}{4}$

Find the measure of the dilation image $S'T'$ or the preimage $ST$ using the given scale factor.

20. $ST = 6, r = -1$
21. $ST = 4, r = \frac{3}{5}$
22. $S'T' = 12, r = \frac{2}{3}$
23. $S'T' = \frac{12}{5}, r = -\frac{3}{5}$
24. $ST = 32, r = -\frac{5}{4}$
25. $ST = 2.25, r = 0.4$
COORDINATE GEOMETRY Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of 2. Then graph a dilation centered at the origin with a scale factor of $\frac{1}{2}$.

26. $F(3, 4), G(6, 10), H(-3, 5)$
27. $X(1, -2), Y(4, -3), Z(6, -1)$
28. $P(1, 2), Q(3, 3), R(3, 5), S(1, 4)$
29. $K(4, 2), L(-4, 6), M(-6, -8), N(6, -10)$

Determine the scale factor for each dilation with center $C$. Determine whether the dilation is an enlargement, reduction, or congruence transformation.

30. $D'$
31. $H'$
32. $O$

33. $D'$
34. $Z'$
35. $E'$

36. **AIRPLANES** Etay is building a model of the SR-71 Blackbird. If the wingspan of his model is 14 inches, what is the approximate scale factor of the model?

PHOTOCOPY For Exercises 37 and 38, refer to the following information.
A 10-inch by 14-inch rectangular design is being reduced on a photocopier by a factor of 75%.
37. What are the new dimensions of the design?
38. How has the area of the preimage changed?

For Exercises 39 and 40, use the following information.
A dilation on a rectangle has a scale factor of 4.
39. What is the effect of the dilation on the perimeter of the rectangle?
40. What is the effect of the dilation on the area of the rectangle?

41. **PROOF** Write a paragraph proof of Theorem 9.1.

42. Triangle $ABC$ has vertices $A(12, 4), B(4, 8),$ and $C(8, -8)$. After two successive dilations centered at the origin with the same scale factor, the final image has vertices $A''(3, 1), B''(1, 2),$ and $C''(2, -2)$. Determine the scale factor $r$ of each dilation from $\triangle ABC$ to $\triangle A''B''C''$.

43. Segment $XY$ has endpoints $X(4, 2)$ and $Y(0, 5)$. After a dilation, the image has endpoints of $X'(7, 17)$ and $Y'(15, 11)$. Find the absolute value of the scale factor.
DIGITAL PHOTOGRAPHY  For Exercises 44–46, use the following information.
Dinah is editing a digital photograph that is 640 pixels wide and 480 pixels high on her monitor.

44. If Dinah zooms the image on her monitor 150%, what are the dimensions of the image?

45. Suppose that Dinah wishes to use the photograph on a web page and wants the image to be 32 pixels wide. What scale factor should she use to reduce the image?

46. Dinah resizes the photograph so that it is 600 pixels high. What scale factor did she use?

47. DESKTOP PUBLISHING  Grace is creating a template for her class newsletter. She has a photograph that is 10 centimeters by 12 centimeters, but the maximum space available for the photograph is 6 centimeters by 8 centimeters. She wants the photograph to be as large as possible on the page. When she uses a scanner to save the photograph, at what percent of the original photograph’s size should she save the image file?

For Exercises 48–50, use quadrilateral $ABCD$.

48. Find the perimeter of quadrilateral $ABCD$.

49. Graph the image of quadrilateral $ABCD$ after a dilation centered at the origin with scale factor $\frac{2}{3}$.

50. Find the perimeter of quadrilateral $A’B’C’D’$ and compare it to the perimeter of quadrilateral $ABCD$.

51. Triangle $TUV$ has vertices $T(6, -5)$, $U(3, -8)$, and $V(-1, -2)$. Find the coordinates of the final image of triangle $TUV$ after a reflection in the x-axis, a translation with $(x, y) \rightarrow (x + 4, y - 1)$, and a dilation centered at the origin with a scale factor of $\frac{1}{3}$. Sketch the preimage and the image.

52. CRITICAL THINKING In order to perform a dilation not centered at the origin, you must first translate all of the points so the center is the origin, dilate the figure, and translate the points back. Consider a triangle with vertices $G(3, 5)$, $H(7, -4)$, and $I(-1, 0)$. State the coordinates of the vertices of the image after a dilation centered at $(3, 5)$ with a scale factor of 2.

53. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do you use dilations when you use a computer?
Include the following in your answer:
• how a “cut and paste” in word processing may be an example of a dilation, and
• other examples of dilations when using a computer.

54. The figure shows two regular pentagons. Find the perimeter of the larger pentagon.

\[ A \quad 5n \quad B \quad 10n \quad C \quad 15n \quad D \quad 60n \]

55. ALGEBRA What is the slope of a line perpendicular to the line given by the equation $3x + 5y = 12$?

\[ A \quad \frac{3}{5} \quad B \quad \frac{5}{3} \quad C \quad -\frac{3}{5} \quad D \quad -\frac{5}{3} \]
Maintain Your Skills

**Mixed Review**

Determine whether a semi-regular tessellation can be created from each figure. Assume that each figure is regular and has a side length of 1 unit. (Lesson 9-4)

56. a triangle and a pentagon  
57. an octagon and a hexagon  
58. a square and a triangle  
59. a hexagon and a dodecagon

**COORDINATE GEOMETRY**

Draw the rotation image of each figure 90° in the given direction about the center point and label the vertices. (Lesson 9-3)

60. \( \triangle ABC \) with \( A(7, -1), B(5, 0), \) and \( C(1, 6) \) counterclockwise about \( P(-1, 4) \)
61. \( \square DEFG \) with \( D(-4, -2), E(-3, 3), F(3, 1), \) and \( G(2, -4) \) clockwise about \( P(-4, -6) \)

62. **CONSTRUCTION**

The Vanamans are building an addition to their house. Ms. Vanaman is cutting an opening for a new window. If she measures to see that the opposite sides are the same length and that the diagonal measures are the same, can Ms. Vanaman be sure that the window opening is rectangular? Explain. (Lesson 8-4)

63. **PREREQUISITE SKILL**

Find \( m \angle A \) to the nearest tenth. (To review finding angles using inverses of trigonometric ratios, see Lesson 7-3.)

64. 65. 66.

**Getting Ready for the Next Lesson**

**Practice Quiz 2**

Lessons 9-4 and 9-5

Determine whether each pattern is a tessellation. If so, describe it as **uniform**, **regular**, **semi-regular**, or **not uniform**. (Lesson 9-4)

1.  
2.  

Draw the dilation image of each figure with center \( C \) and given scale factor. (Lesson 9-5)

3. \( r = \frac{3}{4} \)  
4. \( r = -2 \)  

5. Triangle \( ABC \) has vertices \( A(10, 2), B(1, 6), \) and \( C(-4, 4) \). Find the image of \( \triangle ABC \) after a dilation centered at the origin with scale factor of \( -\frac{1}{2} \). Sketch the preimage and the image. (Lesson 9-5)
Magnitude and Direction

The speed and direction of a plane and the wind can be represented by vectors. A vector in has its initial point at the origin. In the diagram, $\vec{CD}$ is in standard position and can be represented by the ordered pair $\langle 4, 2 \rangle$. A vector can also be drawn anywhere in the coordinate plane. To write such a vector as an ordered pair, find the change in the $x$ values and the change in $y$ values, $(change \ in \ x, change \ in \ y)$, from the tip to the tail of the directed segment. The ordered pair representation of a vector is called the component form of the vector.

A vector in standard position has its initial point at the origin. In the diagram, $\vec{CD}$ is in standard position and can be represented by the ordered pair $(4, 2)$.

A vector can also be drawn anywhere in the coordinate plane. To write such a vector as an ordered pair, find the change in the $x$ values and the change in $y$ values, $(change \ in \ x, change \ in \ y)$, from the tip to the tail of the directed segment. The ordered pair representation of a vector is called the component form of the vector.

Example 1 Write Vectors in Component Form

Write the component form of $\vec{EF}$.

Find the change in $x$-values and the corresponding change in $y$-values.

$\vec{EF} = \langle x_2 - x_1, y_2 - y_1 \rangle$  
Component form of vector

$\quad = \langle 7 - 1, 4 - 5 \rangle$  
$x_1 = 1, y_1 = 5, x_2 = 7, y_2 = 4$

$\quad = \langle 6, -1 \rangle$  
Simplify.

Because the magnitude and direction of a vector are not changed by translation, the vector $\langle 6, -1 \rangle$ represents the same vector as $\vec{EF}$.
The Distance Formula can be used to find the magnitude of a vector. The symbol for the magnitude of \(\overrightarrow{AB}\) is \(|\overrightarrow{AB}|\). The direction of a vector is the measure of the angle that the vector forms with the positive \(x\)-axis or any other horizontal line. You can use the trigonometric ratios to find the direction of a vector.

### Example 2: Magnitude and Direction of a Vector

Find the magnitude and direction of \(\overrightarrow{PQ}\) for \(P(3, 8)\) and \(Q(-4, 2)\).

Find the magnitude.

\[
|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Distance Formula

\[
= \sqrt{(-4 - 3)^2 + (2 - 8)^2}
\]

\[
= \sqrt{49 + 36}
\]

\[
= \sqrt{85}
\]

Simplify.

\[
= 9.2
\]

Use a calculator.

Graph \(\overrightarrow{PQ}\) to determine how to find the direction. Draw a right triangle that has \(\overrightarrow{PQ}\) as its hypotenuse and an acute angle at \(P\).

\[
\tan P = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{2 - 8}{-4 - 3}
\]

\[
= \frac{6}{7}
\]

Simplify.

\[
m\angle P = \tan^{-1} \frac{6}{7}
\]

\[
= 40.6
\]

Use a calculator.

A vector in standard position that is equal to \(\overrightarrow{PQ}\) lies in the third quadrant and forms an angle with the negative \(x\)-axis that has a measure equal to \(m\angle P\). The \(x\)-axis is a straight angle with a measure that is 180. So, the direction of \(\overrightarrow{PQ}\) is \(m\angle P + 180\) or about 220.6°.

Thus, \(\overrightarrow{PQ}\) has a magnitude of about 9.2 units and a direction of about 220.6°.

In Example 1, we stated that a vector in standard position with magnitude of 9.2 units and a direction of 220.6° was equal to \(\overrightarrow{PQ}\). This leads to a definition of equal vectors.

### Key Concept

#### Equal Vectors

Two vectors are equal if and only if they have the same magnitude and direction.

- **Example**: \(\overrightarrow{v} = \overrightarrow{z}\)
- **Nonexample**: \(\overrightarrow{v} \neq \overrightarrow{u}, \overrightarrow{w} \neq \overrightarrow{y}\)

#### Parallel Vectors

Two vectors are parallel if and only if they have the same or opposite direction.

- **Example**: \(\overrightarrow{v} \parallel \overrightarrow{w} \parallel \overrightarrow{z}\)
- **Nonexample**: \(\overrightarrow{v} \parallel \overrightarrow{x}\)
**TRANSLATIONS WITH VECTORS** Vectors can be used to describe translations.

**Example 3** Translations with Vectors

Graph the image of $\triangle ABC$ with vertices $A(-3, -1)$, $B(-1, -2)$, and $C(-3, -3)$ under the translation $\vec{v} = (4, 3)$.

First, graph $\triangle ABC$. Next, translate each vertex by $\vec{v}$, 4 units right and 3 units up. Connect the vertices to form $\triangle A'B'C'$.

Vectors can be combined to perform a composition of translations by adding the vectors. To add vectors, add their corresponding components. The sum of two vectors is called the **resultant**.

**Key Concept**

**Vector Addition**

- **Words** To add two vectors, add the corresponding components.
- **Symbols** If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$, then $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$, and $\vec{b} + \vec{a} = \langle b_1 + a_1, b_2 + a_2 \rangle$.

**Example 4** Add Vectors

Graph the image of $\square QRST$ with vertices $Q(-4, 4)$, $R(-1, 4)$, $S(-2, 2)$, and $T(-5, 2)$ under the translation $\vec{m} = (5, -1)$ and $\vec{n} = (-2, -6)$.

Graph $\square QRST$.

**Method 1** Translate two times.

Translate $\square QRST$ by $\vec{m}$. Then translate the image of $\square QRST$ by $\vec{n}$.

Translate each vertex 5 units right and 1 unit down.
Then translate each vertex 2 units left and 6 units down.
Label the image $\square Q'R'S'T'$.

**Method 2** Find the resultant, and then translate.

Add $\vec{m}$ and $\vec{n}$.

$\vec{m} + \vec{n} = \langle 5 - 2, -1 - 6 \rangle$
$= \langle 3, -7 \rangle$

Translate each vertex 3 units right and 7 units down.
Notice that the vertices for the image are the same for either method.
Comparing Magnitude and Components of Vectors

Model and Analyze
- Draw \( \vec{a} \) in standard position.
- Draw \( \vec{b} \) in standard position with the same direction as \( \vec{a} \), but with a magnitude twice the magnitude of \( \vec{a} \).

1. Write \( \vec{a} \) and \( \vec{b} \) in component form.
2. What do you notice about the components of \( \vec{a} \) and \( \vec{b} \)?
3. Draw \( \vec{b} \) so that its magnitude is three times that of \( \vec{a} \). How do the components of \( \vec{a} \) and \( \vec{b} \) compare?

Make a Conjecture
4. Describe the vector magnitude and direction of a vector \((x, y)\) after the components are multiplied by \( n \).

In the Geometry Activity, you found that a vector can be multiplied by a positive constant, called a scalar, that will change the magnitude of the vector, but not affect its direction. Multiplying a vector by a positive scalar is called scalar multiplication.

Key Concept

Scalar Multiplication

- **Words**
  To multiply a vector by a scalar multiply each component by the scalar.

- **Symbols**
  If \( \vec{a} = (a_1, a_2) \) has a magnitude \( |\vec{a}| \) and direction \( d \), then \( n \vec{a} = n(a_1, a_2) = (na_1, na_2) \), where \( n \) is a positive real number, the magnitude is \( |n \vec{a}| \), and its direction is \( d \).

Example 5

Solve Problems Using Vectors

- **AVIATION** Refer to the application at the beginning of the lesson.

a. Suppose a pilot begins a flight along a path due north flying at 250 miles per hour. If the wind is blowing due east at 20 miles per hour, what is the resultant velocity and direction of the plane?

The initial path of the plane is due north, so a vector representing the path lies on the positive \( y \)-axis 250 units long. The wind is blowing due east, so a vector representing the wind will be parallel to the positive \( x \)-axis 20 units long. The resultant path can be represented by a vector from the initial point of the vector representing the plane to the terminal point of the vector representing the wind.

Use the Pythagorean Theorem.

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
&= 250^2 + 20^2 \\
c^2 &= 62,900 \\
c &= \sqrt{62,900} \\
c &\approx 250.8
\end{align*}
\]

The resultant speed of the plane is about 250.8 miles per hour.

(continued on the next page)
Use the tangent ratio to find the direction of the plane.
\[
\tan \theta = \frac{20}{250} \quad \text{side opposite} = 20, \text{ side adjacent} = 250
\]

\[
\theta = \tan^{-1} \left( \frac{20}{250} \right) \quad \text{Solve for } \theta.
\]

\[
\theta = 4.6 \quad \text{Use a calculator.}
\]

The resultant direction of the plane is about 4.6° east of due north.
Therefore, the resultant vector is 250.8 miles per hour at 4.6° east of due north.

b. If the wind velocity doubles, what is the resultant path and velocity of the plane?

Use scalar multiplication to find the magnitude of the vector for wind velocity.

\[
n | \vec{a} | = 2 | \vec{20} | \quad \text{Magnitude of } n \vec{a}; n = 2, | \vec{a} | = 20
\]

\[
= 2(20) \text{ or } 40 \quad \text{Simplify.}
\]

Next, use the Pythagorean Theorem to find the magnitude of the resultant vector.

\[
c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}
\]

\[
c^2 = 250^2 + 40^2 \quad a = 250, b = 40
\]

\[
c^2 = 64,100 \quad \text{Simplify.}
\]

\[
c = \sqrt{64,100} \quad \text{Take the square root of each side.}
\]

\[
c = 253.2
\]

Then, use the tangent ratio to find the direction of the plane.

\[
\tan \theta = \frac{40}{250} \quad \text{side opposite} = 40, \text{ side adjacent} = 250
\]

\[
\theta = \tan^{-1} \left( \frac{40}{250} \right) \quad \text{Solve for } \theta.
\]

\[
\theta = 9.1 \quad \text{Use a calculator.}
\]

If the wind velocity doubles, the plane flies along a path approximately 9.1° east of due north at about 253.2 miles per hour.

---

**Check for Understanding**

**Concept Check**

1. **OPEN ENDED** Draw a pair of vectors on a coordinate plane. Label each vector in component form and then find their sum.
2. **Compare and contrast** equal vectors and parallel vectors.
3. **Discuss** the similarity of using vectors to translate a figure and using an ordered pair.

**Guided Practice**

4. Write the component form of each vector.

5. Write the component form of each vector.

---

**Reading Math**

The Greek letter theta, \( \theta \), is used to represent the unknown measure of an angle.
Find the magnitude and direction of \( \overrightarrow{AB} \) for the given coordinates.
6. \( A(2, 7), B(-3, 3) \)
7. \( A(-6, 0), B(-12, -4) \)
8. What are the magnitude and direction of \( \overrightarrow{v} = \langle 8, -15 \rangle \)?

Graph the image of each figure under a translation by the given vector.
9. \( \triangle JKL \) with vertices \( J(2, -1), K(-7, -2), L(-2, 8); \overrightarrow{t} = \langle -1, 9 \rangle \)
10. trapezoid \( PQRS \) with vertices \( P(1, 2), Q(7, 3), R(15, 1), S(3, -1); \overrightarrow{u} = \langle 3, -3 \rangle \)
11. Graph the image of \( \square WXYZ \) with vertices \( W(6, -6), X(3, -8), Y(-4, -4), \) and \( Z(-1, -2) \) under the translation by \( \overrightarrow{e} = \langle -1, 6 \rangle \) and \( \overrightarrow{f} = \langle 8, -5 \rangle \).

**Application**
Find the magnitude and direction of each resultant for the given vectors.
12. \( \overrightarrow{g} = \langle 4, 0 \rangle, \overrightarrow{h} = \langle 0, 6 \rangle \)
13. \( \overrightarrow{t} = \langle 0, -9 \rangle, \overrightarrow{u} = \langle 12, -9 \rangle \)
14. **BOATING** Raphael sails his boat due east at a rate of 10 knots. If there is a current of 3 knots moving 30° south of east, what is the resultant speed and direction of the boat?

**Practice and Apply**

Write the component form of each vector.
15. \( \overrightarrow{AB} \) with endpoints \( A(1, -3), B(3, 3) \)
16. \( \overrightarrow{CD} \) with endpoints \( C(-2, 0), D(-3, 4) \)
17. \( \overrightarrow{EF} \) with endpoints \( E(4, 3), F(3, 1) \)
18. \( \overrightarrow{GH} \) with endpoints \( G(-3, 4), H(2, 4) \)
19. \( \overrightarrow{LM} \) with endpoints \( L(4, -2), M(1, 3) \)
20. \( \overrightarrow{OP} \) with endpoints \( O(-1, -1), P(-3, -3) \)

Find the magnitude and direction of \( \overrightarrow{CD} \) for the given coordinates. Round to the nearest tenth.
21. \( C(4, 2), D(9, 2) \)
22. \( C(-2, 1), D(2, 5) \)
23. \( C(-5, 10), D(-3, 6) \)
24. \( C(0, -7), D(-2, -4) \)
25. \( C(-8, -7), D(6, 0) \)
26. \( C(10, -3), D(-2, -2) \)
27. What are the magnitude and direction of \( \overrightarrow{t} = \langle 7, 24 \rangle \)?
28. What are the magnitude and direction of \( \overrightarrow{u} = \langle -12, 15 \rangle \)?
29. What are the magnitude and direction of \( \overrightarrow{v} = \langle -25, -20 \rangle \)?
30. What are the magnitude and direction of \( \overrightarrow{w} = \langle 36, -15 \rangle \)?
Find the magnitude and direction of \( \overrightarrow{MN} \) for the given coordinates. Round to the nearest tenth.

31. \( M(-3, 3), N(-9, 9) \)  
32. \( M(8, 1), N(2, 5) \)  
33. \( M(0, 2), N(-12, -2) \)  
34. \( M(-1, 7), N(6, -8) \)  
35. \( M(-1, 10), N(1, -12) \)  
36. \( M(-4, 0), N(-6, -4) \)

Graph the image of each figure under a translation by the given vector.

37. \( \triangle ABC \) with vertices \( A(3, 6), B(3, -7), C(-6, 1) \); \( \overrightarrow{a} = (0, -6) \)
38. \( \triangle DEF \) with vertices \( D(-12, 6), E(7, 6), F(7, -3) \); \( \overrightarrow{b} = (-3, -9) \)
39. square \( GHIJ \) with vertices \( G(-1, 0), H(-6, -3), I(-9, 2), J(-4, 5) \); \( \overrightarrow{c} = (3, -8) \)
40. quadrilateral \( KLMN \) with vertices \( K(0, 8), L(4, 6), M(3, -3), N(-4, 8) \); \( \overrightarrow{x} = (-10, 2) \)
41. pentagon \( OPQRS \) with vertices \( O(5, 3), P(5, -3), Q(0, -4), R(-5, 0), S(0, 4) \); \( \overrightarrow{y} = (-5, 11) \)
42. hexagon \( TUVWXYZ \) with vertices \( T(4, -2), U(3, 3), V(6, 4), W(9, 3), X(8, -2), Y(6, -5) \); \( \overrightarrow{z} = (-18, 12) \)

Graph the image of each figure under a translation by the given vectors.

43. \( \square ABCD \) with vertices \( A(-1, -6), B(4, -8), C(-3, -11), D(-8, -9) \); \( \overrightarrow{p} = (11, 6), \overrightarrow{q} = (-9, -3) \)
44. \( \triangle XYZ \) with vertices \( X(3, -5), Y(9, 4), Z(12, -2) \); \( \overrightarrow{p} = (2, 2), \overrightarrow{q} = (-4, -7) \)
45. quadrilateral \( EFGH \) with vertices \( E(-7, -2), F(-3, 8), G(4, 15), H(5, -1) \); \( \overrightarrow{p} = (-6, 10), \overrightarrow{q} = (1, -8) \)
46. pentagon \( STUVW \) with vertices \( S(1, 4), T(3, 8), U(6, 8), V(6, 6), W(4, 4) \); \( \overrightarrow{p} = (-4, 5), \overrightarrow{q} = (12, 11) \)

Find the magnitude and direction of each resultant for the given vectors.

47. \( \overrightarrow{a} = (5, 0), \overrightarrow{b} = (0, 12) \)  
48. \( \overrightarrow{c} = (0, -8), \overrightarrow{d} = (-8, 0) \)  
49. \( \overrightarrow{e} = (-4, 0), \overrightarrow{f} = (7, -4) \)  
50. \( \overrightarrow{u} = (12, 6), \overrightarrow{v} = (0, 6) \)  
51. \( \overrightarrow{w} = (5, 6), \overrightarrow{x} = (-1, -4) \)  
52. \( \overrightarrow{y} = (9, -10), \overrightarrow{z} = (-10, -2) \)

53. **SHIPPING** A freighter has to go around an oil spill in the Pacific Ocean. The captain sails due east for 35 miles. Then he turns the ship and heads due south for 28 miles. What is the distance and direction of the ship from its original point of course correction?

54. **RIVERS** Suppose a section of the New River in West Virginia has a current of 2 miles per hour. If a swimmer can swim at a rate of 4.5 miles per hour, how does the current in the New River affect the speed and direction of the swimmer as she tries to swim directly across the river?

**AVIATION** For Exercises 55–57, use the following information.

A jet is flying northwest, and its velocity is represented by \( \langle -450, 450 \rangle \) in miles per hour. The wind is from the west, and its velocity is represented by \( \langle 100, 0 \rangle \) in miles per hour.

55. Find the resultant vector for the jet in component form.
56. Find the magnitude of the resultant.
57. Find the direction of the resultant.
58. CRITICAL THINKING If two vectors have opposite directions but the same magnitude, the resultant is \(0, 0\) when they are added. Find three vectors of equal magnitude, each with its tail at the origin, the sum of which is \(0, 0\).

59. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do vectors help a pilot plan a flight?
Include the following in your answer:
• an explanation of how a wind from the west affects the overall velocity of a plane traveling east, and
• an explanation as to why planes traveling from Hawaii to the continental U.S. take less time than planes traveling from the continental U.S. to Hawaii.

60. If \(\mathbf{q} = \langle 5, 10 \rangle\) and \(\mathbf{r} = \langle 3, 5 \rangle\), find the magnitude for the sum of these two vectors.

\[
\begin{array}{c}
\text{A} & 23 \\
\text{B} & 17 \\
\text{C} & 7 \\
\text{D} & \sqrt{29}
\end{array}
\]

61. ALGEBRA If \(5^b = 125\), then find \(4^b \times 3\).

\[
\begin{array}{c}
\text{A} & 48 \\
\text{B} & 64 \\
\text{C} & 144 \\
\text{D} & 192
\end{array}
\]

Maintain Your Skills

Mixed Review Find the measure of the dilation image \(\mathbf{A'B'}\) or the preimage of \(\mathbf{AB}\) using the given scale factor. (Lesson 9-5)

62. \(\mathbf{AB} = 8, r = 2\)  
63. \(\mathbf{AB} = 12, r = \frac{1}{2}\)  
64. \(\mathbf{A'B'} = 15, r = 3\)  
65. \(\mathbf{A'B'} = 12, r = \frac{1}{4}\)

Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular. (Lesson 9-4)

66.  
67.  

ALGEBRA Use rhombus \(\mathbf{WXYZ}\) with \(m\angle XYZ = 5m\angle WZY\) and \(YZ = 12\). (Lesson 8-5)

68. Find \(m\angle XYZ\).  
69. Find \(WX\).  
70. Find \(m\angle XZY\).  
71. Find \(m\angle WXY\).

72. Each side of a rhombus is 30 centimeters long. One diagonal makes a 25° angle with a side. What is the length of each diagonal to the nearest tenth? (Lesson 7-4)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Perform the indicated operation. (To review operations with matrices, see pages 752–753.)

73. \[
\begin{bmatrix}
-5 & 5 \\
-3 & -2
\end{bmatrix} + 
\begin{bmatrix}
1 & -8 \\
7 & 6
\end{bmatrix}
\]

74. \[
\begin{bmatrix}
-2 & 2 & -2 \\
-7 & -2 & -5
\end{bmatrix} + 
\begin{bmatrix}
-8 & -8 & -8 \\
1 & 1 & 1
\end{bmatrix}
\]

75. \[
3 \begin{bmatrix}
-9 & -5 & -1 \\
9 & 1 & 5
\end{bmatrix}
\]

76. \[
\frac{1}{2} \begin{bmatrix}
-4 & -5 & 0 & 2 \\
4 & 4 & 6 & 0
\end{bmatrix}
\]

77. \[
\begin{bmatrix}
-4 & -4 \\
2 & 2
\end{bmatrix} + 2 \begin{bmatrix}
8 & 4 \\
-3 & -7
\end{bmatrix}
\]

78. \[
\begin{bmatrix}
1 & -1 \\
-1 & 1
\end{bmatrix} \cdot \begin{bmatrix}
2 & -3 \\
-2 & -4
\end{bmatrix}
\]
In Lesson 9-6, you learned that a vector can be represented by the ordered pair \((x, y)\). A vector can also be represented by a column matrix \[
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]. Likewise, polygons can be represented by placing all of the column matrices of the coordinates of the vertices into one matrix, called a vertex matrix.

Triangle \(PQR\) with vertices \(P(3, 5)\), \(Q(1, -2)\), and \(R(-4, 4)\) can be represented by the vertex matrix at the right.

Like vectors, matrices can be used to perform translations. You can use matrix addition and a translation matrix to find the coordinates of a translated figure.

### Example 1 Translate a Figure

Use a matrix to find the coordinates of the vertices of the image of \(\square ABCD\) with \(A(3, 2)\), \(B(1, -3)\), \(C(-3, -1)\), and \(D(-1, 4)\) under the translation \((x, y) \rightarrow (x + 5, y - 3)\).

Write the vertex matrix for \(\square ABCD\):

\[
\begin{bmatrix}
3 & 1 & -3 & -1 \\
2 & -3 & 1 & 4
\end{bmatrix}
\]

To translate the figure 5 units to the right, add 5 to each \(x\)-coordinate. To translate the figure 3 units down, add -3 to each \(y\)-coordinate. This can be done by adding the translation matrix \[
\begin{bmatrix}
5 & 5 & 5 & 5 \\
-3 & -3 & -3 & -3
\end{bmatrix}
\] to the vertex matrix of \(\square ABCD\).

\[
\begin{bmatrix}
3 & 1 & -3 & -1 \\
2 & -3 & 1 & 4
\end{bmatrix}
+ \begin{bmatrix}
5 & 5 & 5 & 5 \\
-3 & -3 & -3 & -3
\end{bmatrix}
= \begin{bmatrix}
8 & 6 & 2 & 4 \\
-1 & -6 & -4 & 1
\end{bmatrix}
\]

The coordinates of \(\square A'B'C'D'\) are \(A'(8, -1)\), \(B'(6, -6)\), \(C'(2, -4)\), and \(D'(4, 1)\).
Scalars can be used with matrices to perform dilations.

**Example 2 Dilate a Figure**

Triangle $FGH$ has vertices $F(-3, 1)$, $G(-1, 2)$, and $H(1, -1)$. Use scalar multiplication to dilate $\triangle FGH$ centered at the origin so that its perimeter is 3 times the original perimeter.

If the perimeter of a figure is 3 times the original perimeter, then the lengths of the sides of the figure will be 3 times the measures of the original lengths. Multiply the vertex matrix by a scale factor of 3.

$$
3 \begin{bmatrix}
-3 & -1 & 1 \\
1 & 2 & -1
\end{bmatrix} = \begin{bmatrix}
-9 & -3 & 3 \\
3 & 6 & -3
\end{bmatrix}
$$

The coordinates of the vertices of $\triangle F'G'H'$ are $F'(-9, 3)$, $G'(-3, 6)$, and $H'(3, -3)$.

**Reflections and Rotations**

A reflection matrix can be used to multiply the vertex matrix of a figure to find the coordinates of the image. The matrices used for four common reflections are shown below.

### Concept Summary

**Reflection Matrices**

<table>
<thead>
<tr>
<th>Reflection in the:</th>
<th>$x$-axis</th>
<th>$y$-axis</th>
<th>origin</th>
<th>line $y = x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply the vertex matrix on the left by:</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{bmatrix}$</td>
</tr>
<tr>
<td>The product of the reflection matrix and the vertex matrix $\begin{bmatrix} a &amp; b \ c &amp; d \end{bmatrix}$ would be:</td>
<td>$\begin{bmatrix} a &amp; b \ -c &amp; -d \end{bmatrix}$</td>
<td>$\begin{bmatrix} -a &amp; -b \ c &amp; d \end{bmatrix}$</td>
<td>$\begin{bmatrix} -a &amp; -b \ -c &amp; -d \end{bmatrix}$</td>
<td>$\begin{bmatrix} c &amp; d \ a &amp; b \end{bmatrix}$</td>
</tr>
</tbody>
</table>

**Example 3 Reflections**

Use a matrix to find the coordinates of the vertices of the image of $\overline{TU}$ with $T(-4, -4)$ and $U(3, 2)$ after a reflection in the $x$-axis.

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for the $x$-axis.

$$
\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 4 & -2 \end{bmatrix}
$$

The coordinates of the vertices of $\overline{T'U'}$ are $T'(-4, 4)$ and $U'(3, -2)$. Graph $\overline{TU}$ and $\overline{T'U'}$.

Matrices can also be used to determine the vertices of a figure’s image by rotation using a rotation matrix. Commonly used rotation matrices are summarized on the next page.
Concept Check

1. Write the reflection matrix for \(\triangle ABC\) and its image \(\triangle A'B'C'\) at the right.

2. Discuss the similarities of using coordinates, vectors, and matrices to translate a figure.

3. OPEN ENDED Graph any \(\square PQRS\) on a coordinate grid. Then write a translation matrix that moves \(\square PQRS\) down and left on the grid.
Guided Practice

Use a matrix to find the coordinates of the vertices of the image of each figure under the given translation.
4. $\triangle ABC$ with $A(5, 4)$, $B(3, -1)$, and $C(0, 2)$; $(x, y) \rightarrow (x - 2, y - 1)$
5. $\square DEFG$ with $D(-1, 3)$, $E(5, 3)$, $F(3, 0)$, and $G(-3, 0)$; $(x, y) \rightarrow (x, y + 6)$

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.
6. $\triangle XYZ$ with $X(3, 4)$, $Y(6, 10)$, and $Z(-3, 5)$; $r = 2$
7. $\square ABCD$ with $A(1, 2)$, $B(3, 3)$, $C(3, 5)$, and $D(1, 4)$; $r = -\frac{1}{4}$

Use a matrix to find the coordinates of the endpoints or vertices of the image of each figure under the given reflection.
8. $\overline{EF}$ with $E(-2, 4)$ and $F(5, 1)$; x-axis
9. quadrilateral $HIJK$ with $H(-5, 4)$, $I(-1, -1)$, $J(-3, -6)$, and $K(-7, -3)$; y-axis

Use a matrix to find the coordinates of the endpoints or vertices of the image of each figure under the given rotation.
10. $\overline{LM}$ with $L(-2, 1)$ and $M(3, 5)$; $90^\circ$ counterclockwise
11. $\triangle PQR$ with $P(6, 3)$, $Q(6, 7)$, and $R(2, 7)$; $270^\circ$ counterclockwise
12. Use matrices to find the coordinates of the image of quadrilateral $STUV$ with $S(-4, 1)$, $T(-2, 2)$, $U(0, 1)$, and $V(-2, -2)$ after a dilation by a scale factor of 2 and a rotation $90^\circ$ counterclockwise about the origin.

Application

LANDSCAPING  For Exercises 13 and 14, use the following information.
A garden design is drawn on a coordinate grid. The original plan shows a rose bed with vertices at $(3, -1)$, $(7, -3)$, $(5, -7)$, and $(1, -5)$. Changes to the plan require that the rose bed’s perimeter be half the original perimeter with respect to the origin, while the shape remains the same.

13. What are the new coordinates for the vertices of the rose bed?
14. If the center of the rose bed was originally located at $(4, -4)$, what will be the coordinates of the center after the changes have been made?

Practice and Apply

Use a matrix to find the coordinates of the endpoints or vertices of the image of each figure under the given translation.
15. $\overline{EF}$ with $E(-4, 1)$, and $F(-1, 3)$; $(x, y) \rightarrow (x - 2, y + 5)$
16. $\triangle JKL$ with $J(-3, 5)$, $K(4, 8)$, and $L(7, 5)$; $(x, y) \rightarrow (x - 3, y - 4)$
17. $\square MNOP$ with $M(-2, 7)$, $N(2, 9)$, $O(2, 7)$, and $P(-2, 5)$; $(x, y) \rightarrow (x + 3, y - 6)$
18. trapezoid $RSTU$ with $R(2, 3)$, $S(6, 2)$, $T(6, -1)$, and $U(-2, 1)$; $(x, y) \rightarrow (x - 6, y - 2)$

Use scalar multiplication to find the coordinates of the vertices of each figure for a dilation centered at the origin with the given scale factor.
19. $\triangle ABC$ with $A(6, 5)$, $B(4, 5)$, and $C(3, 7)$; $r = 2$
20. $\triangle DEF$ with $D(-1, 4)$, $E(0, 1)$, and $F(2, 3)$; $r = -\frac{1}{3}$
21. quadrilateral $GHIJ$ with $G(4, 2)$, $H(-4, 6)$, $I(-6, -8)$, and $J(6, -10)$; $r = -\frac{1}{2}$
22. pentagon $KLMNO$ with $K(1, -2)$, $L(3, -1)$, $M(6, -1)$, $N(4, -3)$, and $O(3, -3)$; $r = 4$
Use a matrix to find the coordinates of the endpoints or vertices of the image of each figure under the given reflection.

23. \( \overline{XY} \) with \( X(2, 2) \) and \( Y(4, -1) \); y-axis

24. \( \triangle ABC \) with \( A(5, -3) \), \( B(0, -5) \), and \( C(-1, -3) \); \( y = x \)

25. quadrilateral \( DEFG \) with \( D(-4, 5) \), \( E(2, 6) \), \( F(3, 1) \), and \( G(-3, -4) \); x-axis

26. quadrilateral \( HIJK \) with \( H(9, -1) \), \( I(2, -6) \), \( J(-4, -3) \), and \( K(-2, 4) \); y-axis

Find the coordinates of the vertices of the image of \( \triangle VWX \) under the given transformation.

27. dilation by scale factor \( \frac{2}{3} \)

28. translation \( (x, y) \rightarrow (x - 4, y - 1) \)

29. rotation 90° counterclockwise about the origin

30. reflection in the line \( y = x \)

Find the coordinates of the vertices of the image of polygon \( PQRST \) under the given transformation.

31. translation \( (x, y) \rightarrow (x + 3, y - 2) \)

32. dilation by scale factor \(-3\)

33. reflection in the y-axis

34. rotation 180° counterclockwise about the origin

Use a matrix to find the coordinates of the endpoints or vertices of the image of each figure under the given rotation.

35. \( \overline{MN} \) with \( M(12, 1) \) and \( N(-3, 10) \); 90° counterclockwise

36. \( \triangle PQR \) with \( P(5, 1) \), \( Q(1, 2) \), and \( R(1, -4) \); 180° counterclockwise

37. \( \square STUV \) with \( S(2, 1) \), \( T(6, 1) \), \( U(5, -3) \), and \( V(1, -3) \); 90° counterclockwise

38. pentagon \( ABCDE \) with \( A(-1, 1) \), \( B(6, 0) \), \( C(4, -8) \), \( D(-4, -10) \), and \( E(-5, -3) \); 270° counterclockwise

Find the coordinates of the image under the stated transformations.

39. dilation by scale factor \( \frac{1}{3} \), then a reflection in the x-axis

40. translation \( (x, y) \rightarrow (x - 5, y + 2) \) then a rotation 90° counterclockwise about the origin

41. reflection in the line \( y = x \) then the translation \( (x, y) \rightarrow (x + 1, y + 4) \)

42. rotation 180° counterclockwise about the origin, then a dilation by scale factor \(-2\)

**PALEONTOLOGY** For Exercises 43 and 44, use the following information.

Paleontologists sometimes discover sets of fossilized dinosaur footprints like those shown at the right.

43. Describe the transformation combination shown.

44. Write two matrix operations that could be used to find the coordinates of point \( C \).
CONSTRUCTION  For Exercises 45 and 46, use the following information.
House builders often use one set of blueprints for many projects. By changing the orientation of a floor plan, a builder can make several different looking houses.

45. Write a transformation matrix that could be used to create a floor plan with the garage on the left.

46. If the current plan is of a house that faces south, write a transformation matrix that could be used to create a floor plan for a house that faces east.

47. CRITICAL THINKING  Write a matrix to represent a reflection in the line $y = -x$.

48. WRITING IN MATH  Answer the question that was posed at the beginning of the lesson.

How can matrices be used to make movies?
Include the following in your answer:
• an explanation of how transformations are used in movie production, and
• an everyday example of transformation that can be modeled using matrices.

49. SHORT RESPONSE  Quadrilateral $ABCD$ is rotated $90^\circ$ clockwise about the origin. Write the transformation matrix.

50. ALGEBRA  A video store stocks 2500 different movie titles. If 26% of the titles are action movies and 14% are comedies, how many are neither action movies nor comedies?
   - A 1000
   - B 1500
   - C 1850
   - D 2150

Maintain Your Skills

Mixed Review  Graph the image of each figure under a translation by the given vector.  

51. $\triangle ABC$ with $A(-6, 1), B(4, 8)$, and $C(1, -4)$; $\vec{v} = (-1, -5)$
52. quadrilateral $DEFG$ with $D(3, -3), E(1, 2), F(8,-1)$, and $G(4, -6)$; $\vec{w} = (-7, 8)$

53. Determine the scale factor used for the dilation at the right, centered at C. Determine whether the dilation is an enlargement, reduction, or congruence transformation.

Find the measures of an exterior angle and an interior angle given the number of sides of a regular polygon.

54. 5
55. 6
56. 8
57. 10

58. FORESTRY  To estimate the height of a tree, Lara sights the top of the tree in a mirror that is 34.5 meters from the tree. The mirror is on the ground and faces upward. Lara is standing 0.75 meter from the mirror, and the distance from her eyes to the ground is 1.75 meters. How tall is the tree?

### Vocabulary and Concept Check

<table>
<thead>
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<th>Term</th>
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<td>rotation</td>
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<td>rotation matrix</td>
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<td>scalar</td>
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<td>transformation</td>
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<tr>
<td>translation matrix</td>
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<td>uniform</td>
<td>(p. 484)</td>
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<tr>
<td>vector</td>
<td>(p. 498)</td>
</tr>
<tr>
<td>vertex matrix</td>
<td>(p. 506)</td>
</tr>
</tbody>
</table>

A complete list of postulates and theorems can be found on pages R1–R8.

### Exercises

State whether each sentence is true or false. If false, replace the underlined word or phrase to make a true sentence.

1. A dilation can change the distance between each point on the figure and the given **line of symmetry**.
2. A tessellation is uniform if the same combination of shapes and angles is present at every vertex.
3. Two vectors can be added easily if you know their **magnitude**.
4. Scalar multiplication affects only the **direction** of a vector.
5. In a rotation, the figure is turned about the **point of symmetry**.
6. A **reflection** is a transformation determined by a figure and a line.
7. A **congruence transformation** is the amount by which a figure is enlarged or reduced in a dilation.
8. A **scalar multiple** is the sum of two other vectors.

### Lesson-by-Lesson Review

**9-1 Reflections**

**Concept Summary**

- The line of symmetry in a figure is a line where the figure could be folded in half so that the two halves match exactly.

**Example**

Copy the figure. Draw the image of the figure under a reflection in line \( \ell \).

The green triangle is the reflected image of the blue triangle.
Exercises  Graph each figure and its image under the given reflection.  
See Example 2 on page 464.
9. triangle ABC with A(2, 1), B(5, 1), and C(2, 3) in the x-axis
10. parallelogram WXYZ with W(–4, 5), X(1, 5), Y(3, 3), and Z(–6, 3) in the line $y = x$
11. rectangle $EFGH$ with $E(–4, –2)$, $F(0, –2)$, $G(0, –4)$, and $H(–4, –4)$ in the line $x = 1$

9-2  
Transl ations

Concept Summary
• A translation moves all points of a figure the same distance in the same direction.
• A translation can be represented as a composition of reflections.

Example  
COORDINATE GEOMETRY  Triangle ABC has vertices A(2, 1), B(4, –2), and C(1, –4). Graph $\triangle ABC$ and its image for the translation $(x, y) \rightarrow (x – 5, y + 3)$.

<table>
<thead>
<tr>
<th>$(x, y)$</th>
<th>$(x – 5, y + 3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 1)</td>
<td>(–3, 4)</td>
</tr>
<tr>
<td>(4, –2)</td>
<td>(–1, 1)</td>
</tr>
<tr>
<td>(1, –4)</td>
<td>(–4, 1)</td>
</tr>
</tbody>
</table>

This translation moved every point of the preimage 5 units left and 3 units up.

Exercises  Graph each figure and the image under the given translation.  
See Example 1 on page 470.
12. $\square EFGH$ with $E(2, 2)$, $F(6, 2)$, $G(4, –2)$, $H(1, –1)$ under the translation $(x, y) \rightarrow (x – 4, y – 4)$
13. $\overline{ST}$ with endpoints $S(–3, –5), T(–1, –1)$ under the translation $(x, y) \rightarrow (x + 2, y + 4)$
14. $\triangle XYZ$ with $X(2, 5)$, $Y(1, 1)$, $Z(5, 1)$ under the translation $(x, y) \rightarrow (x + 1, y – 3)$

• A rotation turns each point in a figure through the same angle about a fixed point.
• An object has rotational symmetry when you can rotate it less than $360^\circ$ and the preimage and image are indistinguishable.

Example  Identify the order and magnitude of the rotational symmetry in the figure.
The figure has rotational symmetry of order 12 because there are 12 rotations of less than $360^\circ$ (including $0^\circ$) that produce an image indistinguishable from the original.
The magnitude is $360^\circ \div 12$ or $30^\circ$.
Exercises  Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.  See Example 2 on page 478.

15. $\triangle BCD$ with vertices $B(-3, 5)$, $C(-3, 3)$, and $D(-5, 3)$ reflected in the $x$-axis and then the $y$-axis

16. $\triangle FGH$ with vertices $F(0, 3)$, $G(-1, 0)$, $H(-4, 1)$ reflected in the line $y = x$ and then the line $y = -x$

17. $\triangle LMN$ with vertices $L(2, 2)$, $M(5, 3)$, $N(3, 6)$ reflected in the line $y = -x$ and then the $x$-axis

The figure at the right is a regular nonagon.  See Exercise 3 on page 478.

18. Identify the order and magnitude of the symmetry.

19. What is the measure of the angle of rotation if vertex 2 is moved counterclockwise to the current position of vertex 6?

20. If vertex 5 is rotated $280^\circ$ counterclockwise, find its new position.

Tessellations

9-4

Concept Summary

- A tessellation is a repetitious pattern that covers a plane without overlap.
- A regular tessellation contains the same combination of shapes and angles at every vertex.

Example

Classify the tessellation at the right.

The tessellation is uniform, because at each vertex there are two squares and three equilateral triangles. Both the square and equilateral triangle are regular polygons.

Since there is more than one regular polygon in the tessellation, it is a semi-regular tessellation.

Exercises  Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.  See Example 3 on page 485.

21.  

22.  

23.  

Exercises  Determine whether each regular polygon will tessellate the plane. Explain.  See Example 1 on page 484.

24. pentagon  

25. triangle  

26. decagon
9-5  

**Dilations**

**Concept Summary**
- Dilations can be enlargements, reductions, or congruence transformations.

**Example**

Triangle $EFG$ has vertices $E(-4, -2), F(-3, 2),$ and $G(1, 1)$. Find the image of $\triangle EFG$ after a dilation centered at the origin with a scale factor of $\frac{3}{2}$.

<table>
<thead>
<tr>
<th>Preimage $(x, y)$</th>
<th>Image $(\frac{3}{2}x, \frac{3}{2}y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(-4, -2)$</td>
<td>$E'(-6, -3)$</td>
</tr>
<tr>
<td>$F(-3, 2)$</td>
<td>$F'(-\frac{9}{2}, 3)$</td>
</tr>
<tr>
<td>$G(1, 1)$</td>
<td>$G'\left(\frac{3}{2}, \frac{3}{2}\right)$</td>
</tr>
</tbody>
</table>

**Exercises**

Find the measure of the dilation image $C'D'$ or preimage of $CD$ using the given scale factor.  
See Example 1 on page 491.

27. $CD = 8, r = 3$  
28. $CD = \frac{2}{3}, r = -6$  
29. $C'D' = 24, r = 6$

30. $C'D' = 60, r = \frac{10}{3}$  
31. $CD = 12, r = \frac{5}{6}$  
32. $C'D' = \frac{55}{2}, r = \frac{5}{4}$

Find the image of each polygon, given the vertices, after a dilation centered at the origin with a scale factor of $-2$. See Example 3 on page 492.

33. $P(-1, 3), Q(2, 2), R(1, -1)$  
34. $E(-3, 2), F(1, 2), G(1, -2), H(-3, -2)$

9-6  

**Vectors**

**Concept Summary**
- A vector is a quantity with both magnitude and direction.
- Vectors can be used to translate figures on the coordinate plane.

**Example**

Find the magnitude and direction of $\overline{PQ}$ for $P(-8, 4)$ and $Q(6, 10)$.

Find the magnitude.

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(6 + 8)^2 + (10 - 4)^2}$$
$$= \sqrt{152}$$
$$\approx 15.2$$

Find the direction.

$$\tan P = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$
$$= \frac{10 - 4}{6 + 8}$$
$$= \frac{6}{14} \text{ or } \frac{3}{7}$$

$$m \angle P = \tan^{-1} \frac{3}{7}$$
$$\approx 23.2$$

Use a calculator.
Exercises  Write the component form of each vector.  See Example 1 on page 498.

35. \( \vec{A} \) \((-3, -2) \), \( \vec{B} \) \((0, 2) \)

36. \( \vec{D} \) \((-4, 2) \), \( \vec{O} \) \((4, -2) \)

37. \( \vec{F} \) \((1, 4) \), \( \vec{E} \) \((1, -4) \)

Find the magnitude and direction of \( \vec{AB} \) for the given coordinates.  See Example 2 on page 499.

38. \( A(-6, 4), B(-9, -3) \)

39. \( A(8, 5), B(-5, -2) \)

40. \( A(-14, 2), B(15, -5) \)

41. \( A(16, 40), B(45, 0) \)

9-7 Transformations with Matrices

Concept Summary
- The vertices of a polygon can be represented by a vertex matrix.
- Matrix operations can be used to perform transformations.

Example
Use a matrix to find the coordinates of the vertices of the image of \( \triangle ABC \) with \( A(1, -1), B(2, -4), C(7, -1) \) after a reflection in the \( y \)-axis.

Write the ordered pairs in a vertex matrix. Then use a calculator to multiply the vertex matrix by the reflection matrix.

\[
\begin{bmatrix}
-1 & 0 \\
0 & 1 
\end{bmatrix} \begin{bmatrix}
1 & 2 & 7 \\
-1 & -4 & -1 
\end{bmatrix} = \begin{bmatrix}
-1 & -2 & -7 \\
-1 & -4 & -1 
\end{bmatrix}
\]

The coordinates of \( \triangle A'B'C' \) are \( A'(-1, -1) \), \( B'(-2, -4) \), and \( C'(-7, -1) \).

Exercises  Use a matrix to find the coordinates of the vertices of the image after the stated transformation.  See Example 1 on page 506.

42. translation \((x, y) \rightarrow (x - 3, y - 6) \)

43. dilation by scale factor \( \frac{4}{5} \)

44. reflection in the line \( y = x \)

45. rotation 270° counterclockwise about the origin

Find the coordinates of the image after the stated transformations.  See Examples 2–4 on pages 507 and 508.

46. \( \triangle PQR \) with \( P(9, 2), Q(1, -1), \) and \( R(4, 5) \); \((x, y) \rightarrow (x + 2, y - 5) \), then a reflection in the \( x \)-axis

47. quadrilateral \( WXYZ \) with \( W(-8, 1), X(-2, 3), Y(-1, 0), \) and \( Z(-6, -3) \); a rotation 180° counterclockwise, then a dilation by scale factor \(-2\)
Choose the correct term to complete each sentence.
1. If a dilation does not change the size of the object, then it is a(n) (isometry, reflection).
2. Tessellations with the same shapes and angles at each vertex are called (uniform, regular).
3. A vector multiplied by a (vector, scalar) results in another vector.

Name the reflected image of each figure under a reflection in line \(m\).
4. \(A\)
5. \(\overline{BC}\)
6. \(\triangle DCE\)

COORDINATE GEOMETRY
Graph each figure and its image under the given translation.
7. \(\triangle PQR\) with \(P(-3, 5), Q(-2, 1),\) and \(R(-4, 2)\) under the translation right 3 units and up 1 unit
8. Parallelogram \(WXYZ\) with \(W(-2, -5), X(1, -5), Y(2, -2),\) and \(Z(-1, -2)\) under the translation up 5 units and left 3 units
9. \(\overline{FG}\) with \(F(3, 5)\) and \(G(6, -1)\) under the translation \((x, y) \rightarrow (x - 4, y - 1)\)

Draw the rotation image of each triangle by reflecting the triangles in the given lines. State the coordinates of the rotation image and the angle of rotation.
10. \(\triangle JKL\) with \(J(-1, -2), K(-3, -4),\) and \(L(1, -4)\) reflected in the \(y\)-axis and then the \(x\)-axis
11. \(\triangle ABC\) with \(A(-3, -2), B(-1, 1),\) and \(C(3, -1)\) reflected in the line \(y = x\) and then the line \(y = -x\)
12. \(\triangle RST\) with \(R(1, 6), S(1, 1),\) and \(T(3, -2)\) reflected in the \(y\)-axis and then the line \(y = x\)

Determine whether each pattern is a tessellation. If so, describe it as uniform, not uniform, regular, or semi-regular.
13. 
14. 
15. 

Find the measure of the dilation image \(\overline{M'N'}\) or preimage of \(\overline{MN}\) using the given scale factor.
16. \(MN = 5, r = 4\)
17. \(MN = 8, r = \frac{1}{4}\)
18. \(M'N' = 36, r = 3\)
19. \(MN = 9, r = -\frac{1}{5}\)
20. \(M'N' = 20, r = \frac{2}{3}\)
21. \(M'N' = \frac{29}{5}, r = -\frac{3}{5}\)

Find the magnitude and direction of each vector.
22. \(\overrightarrow{v} = (-3, 2)\)
23. \(\overrightarrow{w} = (-6, -8)\)

TRAVEL
In trying to calculate how far she must travel for an appointment, Gunja measured the distance between Richmond, Virginia, and Charlotte, North Carolina, on a map. The distance on the map was 2.25 inches, and the scale factor was 1 inch equals 150 miles. How far must she travel?

STANDARDIZED TEST PRACTICE
What reflections could be used to create the image \((3, 4)\) from \((3, -4)\)?
I. reflection in the \(x\)-axis
II. reflection in the \(y\)-axis
III. reflection in the origin
A. I only
B. III only
C. I and III
D. I and II
1. Ms. Lee told her students, “If you do not get enough rest, you will be tired. If you are tired, you will not be able to concentrate.” Which of the following is a logical conclusion that could follow Ms. Lee’s statements? (Lesson 2-4)

A. If you get enough rest, you will be tired.
B. If you are tired, you will be able to concentrate.
C. If you do not get enough rest, you will be able to concentrate.
D. If you do not get enough rest, you will not be able to concentrate.

2. Which of the following statements is true? (Lesson 3-5)

A. \(\overline{CE} \parallel \overline{DF}\)
B. \(\overline{CF} \parallel \overline{DG}\)
C. \(\overline{CF} \cong \overline{DF}\)
D. \(\angle CE \cong \angle DF\)

3. Which of the following would not prove that quadrilateral \(QRST\) is a parallelogram? (Lesson 8-2)

A. Both pairs of opposite angles are congruent.
B. Both pairs of opposite sides are parallel.
C. Diagonals bisect each other.
D. A pair of opposite sides is congruent.

4. If \(Q(4, 2)\) is reflected in the \(y\)-axis, what will be the coordinates of \(Q’\)? (Lesson 9-1)

A. \((-4, -2)\)
B. \((-4, 2)\)
C. \((2, -4)\)
D. \((2, 4)\)

5. Which of the following statements about the figures below is true? (Lesson 9-2)

A. Parallelogram \(JKLM\) is a reflection image of \(\square ABCD\).
B. Parallelogram \(EFGH\) is a translation image of \(\square ABCD\).
C. Parallelogram \(JKLM\) is a translation image of \(\square EFGH\).
D. Parallelogram \(JKLM\) is a translation image of \(\square ABCD\).

6. Which of the following is not necessarily preserved in a congruence transformation? (Lesson 9-2)

A. angle and distance measure
B. orientation
C. collinearity
D. betweenness of points

7. Which transformation is used to map \(\triangle ABC\) to \(\triangle A’B’C’\)? (Lesson 9-3)

A. rotation
B. reflection
C. dilation
D. translation
Part 2  Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. A new logo was designed for GEO Company. The logo is shaped like a symmetrical hexagon. What are the coordinates of the missing vertex of the logo? (Lesson 1-1)

9. A soccer coach is having her players practice penalty kicks. She places two cones equidistant from the goal and asks the players to line up behind each cone. What is the value of \( x \)? (Lesson 4-6)

10. A steel cable, which supports a tram, needs to be replaced. To determine the length \( x \) of the cable currently in use, the engineer makes several measurements and draws the diagram below of two right triangles, \( \triangle ABC \) and \( \triangle EDC \). If \( m \angle ACB = m \angle ECD \), what is the length \( x \) of the cable currently in use? Round the result to the nearest meter. (Lesson 6-3)

Part 3  Extended Response

Record your answers on a sheet of paper. Show your work.

11. Kelli drew the diagram below to show the front view of a circus tent. Prove that \( \triangle ABD \) is congruent to \( \triangle ACE \). (Lessons 4-5 and 4-6)

12. Paul is studying to become a landscape architect. He drew a map view of a park with the following vertices: \( Q(2, 2) \), \( R(-2, 4) \), \( S(-3, -2) \), and \( T(3, -4) \)

a. On a coordinate plane, graph quadrilateral \( QRST \). (Prerequisite Skill)

b. Paul’s original drawing appears small on his paper. His instructor says that he should dilate the image with the origin as center and a scale factor of 2. Graph and label the coordinates of the dilation image \( Q'R'S'T' \). (Lesson 9-5)

c. Explain how Paul can determine the coordinates of the vertices of \( Q'R'S'T' \) without using a coordinate plane. Use one of the vertices for a demonstration of your method. (Lesson 9-5)

d. Dilations are similarity transformations. What properties are preserved during an enlargement? reduction? congruence transformation? (Lesson 9-5)