Planetary and Satellite Motion - Kepler’s Laws

1. Johannes Kepler (German 1618)
   a. Student of Tycho Brahe, Danish astronomer
   b. Analyzed Brahe’s data, focusing on Mars.

2. 3 Laws that describe the behavior of all planets and satellites
   a. Law of Ellipses
      i. all planets follow elliptical paths with the Sun at one focus
   b. Law of Areas
      i. An imaginary line drawn from the sun to a planet sweeps out equal areas (A & B) in equal time intervals (t₁ - t₂ and t₃ - t₄). Thus planets move fastest when they are closest to the sun (perigee) and slowest when they are furthest from the sun (apogee).
      ii. The law of “Conservation of Angular Momentum (L)” applies for the orbital motion of the planet (or satellite) in that at all points in the orbit the product of the velocity (v) and orbital radius (r) is constant, per 2.b.i above.
         1. at an orbital point 1, the angular momentum (L₁) = mass * v₁ * r₁
         2. at an orbital point 2, the angular momentum (L₂) = mass * v₂ * r₂
         3. L₁ = L₂ (conservation of angular momentum)
         4. mass * v₁ * r₁ = mass * v₂ * r₂ (cancel masses), then
            5. v₁r₁ = v₂r₂
   c. Laws of Periods/Harmonies
      i. The square of the orbital period (T) of a planet is proportional to the cube of the average orbital radius (R).
ii. \( T^2 \propto R^3 \), or

iii. \( \frac{T^2}{R^3} = \text{constant} = \frac{4\pi^2}{GM} \)

where

1. the constant of proportionality \((4\pi^2/GM)\) for planets around the sun = 2.9 \( \times 10^{-19} \)

3. Conventions

a. Astronomical Unit (AU) - length

i. The distance from the earth to the sun = \(1.49 \times 10^{11}\) meters = \textbf{1 AU}

b. Earth Years (EY) - time

i. The period for the earth to orbit the sun = \(3.156 \times 10^7\) secs = \textbf{1 EY}

c. Then the constant of proportionality for Kepler’s 3\textsuperscript{rd} Law becomes 1, and

i. \( \frac{T^2}{R^3} = 1 \), or

ii. \( T^2 = R^3 \)

d. Using these conventions, that \textbf{compare} a planet’s period and orbital radius to the earth’s period and orbital radius, calculations relating to planetary characteristics of period and orbital radius are much simpler, as they are expressed as a multiplier of the earth’s characteristics.