Simple Harmonic Motion (SHM)

Overview
(Ref p368)
Terminology for Periodic Motion

- **Period** (T)
  - The time, in seconds, it takes for a vibrating object to repeat its motion – seconds per vibration, oscillation or cycle

- **Frequency** (f)
  - The number of vibrations made per unit time – vibration, oscillation or cycles per second (Hz)

- **T = 1/f**
  - The relationship is reciprocal

- **Amplitude** (A or x)
  - The displacement from rest position
SHM - Description

An object is said to be in simple harmonic motion if the following occurs:

• It moves in a **uniform** path.
• A variable **force** acts on it.
• The magnitude of force is **proportional** to the displacement of the mass.
• The force is always **opposite** in direction to the displacement direction.
• The motion is **repetitive** and a round trip, back and forth, is always made in equal time periods.
SHM Visually

- **Examples**
  - Spring
  - Pendulum
**SHM – Hooke’s Law**

- SHM describes any **periodic** motion that results from a **restoring** force \( F \) that is proportional to the **displacement** \( x \) of an object from its equilibrium position.

\[
F_{\text{rest}} = -kx, \text{ where } k = \text{ spring constant}
\]

- **Note:**
  - **Elastic limit** – if exceeded, the spring does **not** return to its original shape
  - Law applies equally to horizontal and vertical models
Hooke’s Law – Horizontal Springs

- At max displacement (2 & 4), spring force and acceleration reach a maximum and velocity (thus KE) is zero.
- At zero displacement (1 & 3) PE is zero, thus KE and velocity are maximum.
- The larger the k value the stiffer the spring.
- Negative sign indicates the restoring force is opposite the displacement.
Hooke’s Law – Vertical Springs

- Hooke’s Law applies equally to a vertical model of spring motion, in which the weight of the mass provides a force.

- @ Equilibrium position with no motion:
  - Spring force↑ = weight↓
Practice

- A load of 50 N stretches a vertical spring by 0.15 m. What is the spring constant?
- Solve $F = -kx$ for $k$
  - $50 = -k \times 0.15$
  - $k = -50/0.15 = 333.3$ N/m (drop the – sign)
Mass-Spring System - Period

- The period of a mass-spring can be calculated as follows:

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

\[ T = \frac{2\pi}{\sqrt{k \text{ mass const} \text{ spring}}} \]
Practice

- What is the spring constant of a mass-spring system that has a mass of 0.40 kg and oscillates with a period of 0.2 secs?

- Solve \( T = 2\pi \sqrt{\frac{m}{k}} \) for \( k \)

  \[
  0.2 = 2\pi \sqrt{0.4/k} 
  \]

  - \( k = 394.8 \) N/m
If a mass of 0.55 kg stretches a vertical spring 2 cm from its rest position, what is the spring constant (k)?

Solve $F = -kx$ for $k$ (or $\Delta F = -k*\Delta x$)

- $k = F/x$, where $F = \text{weight (mg)}$ of the mass
- $k = mg/x = 0.55 \times 9.8/0.02$
- $k = 269.5 \text{ N/m}$
If a pendulum of length $l$ is disturbed through an angle $\theta$ (1 or 3), the restoring force ($F$) component drives the bob back (and through) the rest (2) position.

$$\text{Period}(T) = 2\pi \sqrt{\frac{\text{length}}{\text{grav acc}}} = 2\pi \sqrt{\frac{l}{g}}$$
Practice - pendulum

- What period would you expect from a pendulum of length 0.5 m on the moon where \( g = 1.6 \text{ m/s}^2 \)?

- Solve \( T = 2\pi \sqrt{\frac{l}{g}} \)
  - \( T = 2\pi \sqrt{(0.5/1.6)} \)
  - \( T = 3.51 \text{ seconds} \)
Practice - pendulum

- What is the frequency \( f \) of a 3 m \( l \) swing at the North Pole, where \( g = 9.83 \) m/s\(^2\)?

- Solve \( T = 2\pi \sqrt{\frac{l}{g}} \)
  - \( T = 2\pi \sqrt{\frac{3}{9.83}} \)
  - \( T = 3.47 \) sec, therefore…
  - \( f = \frac{1}{T} = 0.29 \) Hz
Energy Considerations for SHM

- **Springs**
  - $\text{PE max}$ at maximum displacement
  - $\text{KE max}$ while passing through the rest position
Spring Energy Summary

\[ x = 0 \]

\[ F_{net} = 0 \]
\[ v = 0 \]
\[ KE = 0 \]
\[ PE = max \]

\[ F_{net} = \text{max up} \]
\[ v = 0 \]
\[ KE = 0 \]
\[ PE = max \]

\[ x_{max} \]

\[ d \]

\[ F_{net} \]
\[ v = \text{max} \]
\[ KE = \text{max} \]
\[ PE = 0 \]

\[ x_{min} \]

\[ d \]

\[ F_{net} \]
\[ v = \text{max} \]
\[ KE = \text{max} \]
\[ PE = 0 \]

\[ x_{max} \]

\[ F_{net} = \text{max up} \]
\[ v = 0 \]
\[ KE = 0 \]
\[ PE = max \]
Pendulums

- **PE max** at maximum disturbance angle
- **KE max** at bottom of arc
Pendulum Energy summary

\[ v = 0 \quad PE = \text{max} \]
\[ F_r = \text{max} \quad KE = 0 \]

\[ \theta = -\theta_{\text{max}} \]

\[ v = 0 \quad PE = \text{max} \]
\[ F_r = \text{max} \quad KE = 0 \]

\[ \theta = 0 \]

\[ v = \text{max} \quad PE = 0 \]
\[ F_r = 0 \quad KE = \text{max} \]

\[ \theta = \theta_{\text{max}} \]
Summary formulas

- **Period (T)** = 1/frequency (f)
  - $T = \frac{1}{f}$

- **Hooke’s Law**
  - Force = spring const $\times$ displacement
  - $F = kx$ (drop negative sign)

- **Spring period**
  - $T = 2\pi\sqrt{\frac{m}{k}}$

- **Pendulum**
  - $T = 2\pi\sqrt{\frac{l}{g}}$