It has rained for the past three days. However, the weather forecast for tomorrow's football game is sunny and no chance of rain. You decide not to take a rain jacket, only to get drenched when there is a sudden downpour during the game.

Although not perfect, the level of accuracy of weather forecasting has increased significantly through the use of computer models. These models analyze current data and predict atmospheric conditions at some short period of time from that moment. Based on these predicted conditions, another set of atmospheric conditions are predicted. This process continues until the forecast for the day and the extended forecast for the next several days are completed.

1. a. What does the level of accuracy in the weather forecasting model described above depend upon?

b. The level of accuracy decreases as the forecasts extend several days ahead. Explain.

The accuracy of weather forecasting is based upon conditional probabilities; that is, the probability that one event happens, given that another event has occurred. For example, the probability that it will rain this afternoon, given that a low pressure system moved into the area this morning, is a conditional probability.

Weather forecasting is revisited in Situation 2 of Activity 6.5.

Calculating Conditional Probabilities

How do you determine conditional probabilities? First, consider the following problem to see how the condition affects the sample space.

2. A family with two children is moving into your neighborhood.

a. Determine the sample space of possibilities for the gender of the first and second child using a tree diagram.

b. Assume that the probability of a boy and girl are each \( \frac{1}{2} \). Are the outcomes in the sample space equally likely? Explain.
c. What is the probability that both children are boys?

d. What is the probability that both children are boys, given that the first child is a boy?

In Problem 2c, you selected outcomes from the entire sample space for which the event (both boys) was true. In Problem 2d, you are given that the first child is a boy. Therefore, the sample space is reduced to only include outcomes that make the given event true. The outcomes in which the first child is not a boy are neglected.

The probability that both children are boys, given that the first child is a boy, is a conditional probability. There is a special notation for conditional probabilities.

In general, \( P(A|B) \) denotes the probability of event \( A \), given that event \( B \) has occurred.

In Problem 2d, the probability that both children are boys, given that the first child is a boy, is written as

\[
P(\text{both boys} \mid \text{first is a boy})
\]

3. Referring to Problem 2, what is the probability that both children are boys, given at least one of the children is a boy?

Calculating Conditional Probabilities Using a Formula

In general, a formula is used to calculate conditional probabilities. The following gives an intuitive justification for this formula.

4. Suppose a person is selected at random in the United States. Now, let

\( A \) represent the event that the person selected is a student at the University of Virginia, and  
\( B \) represent the event that the person selected is a resident of Charlottesville, VA.

a. What can you say about \( P(A) \) and \( P(B) \)? Explain.
b. How does the \( P(A) \) in part a compare to the probability of attending the University of Virginia given that the person selected lives in Charlottesville, VA? That is, how does \( P(A) \) in part a compare to \( P(A|B) \)? Explain.

The situation in Problem 4 can be demonstrated by the following Venn diagram. All the people who live in the United States are contained in the rectangle; all the people who attend the University of Virginia are contained in circle A; all people who live in Charlottesville, VA are in circle B.

5. a. Describe the region that represents the new sample space, given that the person selected lives in Charlottesville.

b. Describe the region that represents the outcomes that satisfy the event that the person selected is a student at the University of Virginia, given the person lives in Charlottesville.

Therefore, the \( P(A|B) \) should be a fraction of the number of people that are both a student at Virginia and live in Charlottesville divided by the number of people who live in Charlottesville. Stated symbolically,

\[
P(A|B) = \frac{n(A \text{ and } B)}{n(B)},
\]

where \( n(A \text{ and } B) \) represents the number of outcomes common to both events \( A \) and \( B \), and \( n(B) \) is the number of outcomes for which event \( B \) is true. Note that \( n(B) \neq 0 \).

6. In Problem 2d, you used the reduced sample space to determine the probability that the two children are both boys, given the first child is a boy. Redo the problem using the conditional probability formula. Compare the results.
7. A taste test is given to 200 people at a local mall. The results are summarized in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Prefers Coke</th>
<th>Prefers Pepsi</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>51</td>
<td>40</td>
<td>91</td>
</tr>
<tr>
<td>Female</td>
<td>34</td>
<td>75</td>
<td>109</td>
</tr>
<tr>
<td>Total</td>
<td>85</td>
<td>115</td>
<td>200</td>
</tr>
</tbody>
</table>

If one person who participated in the taste test is randomly selected, use the data in the table to determine the probability that the person selected

a. prefers Coke.  
b. is a male.

c. is a male and prefers Coke.

d. prefers Coke, given that a male is selected.

e. prefers Pepsi, given that a female is selected.

In many applications, you don’t know the actual numbers, but you do know the percentages and probabilities. In such situations, the formula used to calculate conditional probabilities is

\[
P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}, \text{ where } P(B) \neq 0.
\]

The probabilities \(P(A \text{ and } B)\) and \(P(B)\) are determined using the original sample space.

8. In Europe, 88% of all households have a television and 51% of all households have a television and a VCR.

a. If a household in Europe is randomly selected, what is the probability that the household has a television?

b. Determine the probability that a randomly selected household has a television and a VCR.
c. What is the probability that a household has a VCR given that it has a television?

9. Motor vehicles sold in the United States are classified as either cars or light trucks, and as either domestic or imported. Light trucks include SUVs and minivans. The following table gives the probabilities for a randomly selected vehicle recently sold in the United States. For example, the probability of selecting a domestic light truck is 0.47.

<table>
<thead>
<tr>
<th></th>
<th>DOMESTIC</th>
<th>IMPORTED</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light truck</td>
<td>0.47</td>
<td>0.07</td>
<td>0.54</td>
</tr>
<tr>
<td>Car</td>
<td>0.33</td>
<td>0.13</td>
<td>0.46</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

a. Complete the table.

In parts b–g, determine the probability that a randomly selected vehicle sold in the United States is

b. a car.

c. imported.

d. imported car.

e. a car or imported.

f. a car, given that the vehicle is imported.

g. light truck, given that the vehicle is domestic.